

Production, Manufacturing and Logistics

Selling to the “Newsvendor” with a forecast update:
Analysis of a dual purchase contract ☆,☆☆

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Abstract

We consider a supply chain in which a manufacturer sells to a *procure-to-stock* retailer facing a newsvendor problem with a forecast update. Under a wholesale price contract, the retailer waits as long as she can and optimally places her order *after* observing the forecast update. We show that the retailer’s wait-and-decide strategy, induced by the wholesale price contract, hinders the manufacturer’s ability to (1) set the wholesale price and maximize his profit, (2) hedge against excess inventory risk, and (3) reduce his profit uncertainty. To mitigate the adverse effect of wholesale price contract, we propose the *dual purchase* contract, through which the manufacturer provides a discount for orders placed before the forecast update. We characterize how and when a dual purchase contract creates strict Pareto improvement over a wholesale price contract. To do so, we establish the retailer’s optimal ordering policy and the manufacturer’s optimal pricing and production policies. We show how the dual purchase contract reduces profit variability and how it can be used as a risk hedging tool for a risk averse manufacturer. Through a numerical study, we provide additional managerial insights and show, for example, that market uncertainty is a key factor that defines when the dual purchase contract provides strict Pareto improvement over the wholesale price contract.

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1. Introduction

In many supply chains, the upstream firm (the manufacturer) must adjust to the downstream firm’s (the retailer’s) demand through continuous refinement of production processes and delivery systems. A well-known example from the apparel industry is *quick response*, a series of process improvement initiatives that enable

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faster response to retailer orders. Such initiatives allow the retailer to wait until the last possible minute, placing her order after a better forecast becomes available. The practice of last minute ordering is not unique to the apparel industry. A 2002 survey shows that 57% of industrial buyers from automotive to electronics increased “just-in-time” purchases and reduced buying based on long term forecasts (Ansberry, 2002).

In this paper, we show that last minute ordering and its adverse effect on manufacturers are the result of a wholesale price contract, which is widely used due to its simplicity. Often the procurement contract between a manufacturer and a retailer is negotiated well in advance of the sales season, actual orders and product delivery. Last minute ordering coupled with early commitment to a wholesale price further squeezes the manufacturer’s profit. Ansberry (2002, Wall Street Journal) reports anecdotal evidence to conclude that

“... [retailers] are holding onto their cash as long as they can ... waiting until the last possible moment to ensure that every order they place will lead to profits ... [those] manufacturers receiving last minute orders have difficulty in justifying equipment purchases. many manufacturers invest in expensive equipment ... to justify the cost is to run [the equipment] constantly ... the system works best when [retailer] places big orders well in advance ...”

The above observations suggest that the consequence of last minute ordering (due to the wholesale price contract) to the manufacturer is threefold. First, the manufacturer may not be able to charge the wholesale price that maximizes his profit. The negotiation to set the wholesale price often takes place well in advance of the sales season. Hence, the market potential for the product is uncertain during this negotiation. This uncertainty hinders the manufacturer’s ability to enforce the optimal wholesale price. Second, last minute ordering negatively affects the manufacturer’s ability to invest in new technology or justify equipment purchases. Under a wholesale price contract, the retailer delays her ordering decision as much as she can and does not commit to buying any product prior to obtaining a final forecast update, hence better demand information. The retailer allows just enough time to the manufacturer to produce her orders. The retailer’s *wait-and-see* strategy, however, requires the manufacturer to invest in production equipment in the face of uncertain profits. Third, the manufacturer often initiates part of his production prior to receiving the retailer’s order when his costs are decreasing in production lead time. In this case, the manufacturer also faces excess inventory risk.

In this paper, we provide a new contract form with which the manufacturer can (1) push inventory to the retailer, known also as *channel stuffing*, (2) create a strict Pareto improvement over the wholesale price contract while inheriting the wholesale price contract’s simplicity, and (3) reduce the manufacturer’s profit variability. To do so, we propose a *dual purchase* contract that induces a retailer to place two consecutive orders; before and after obtaining the final forecast update. A dual purchase contract specifies two prices: a per unit advance purchase price w_a for orders placed prior to the forecast update and the per unit wholesale price w for orders placed after the forecast update (and hence closer to the sales season). The retailer often obtains the forecast update after a major trade show conducted close to the sales season as in the apparel industry (see, for example, Zara case study¹). In this case, the advance purchase price can be charged for each unit ordered prior to this trade show.

First, we study the wholesale price contract with which a *procure-to-stock* retailer pays a manufacturer w per unit ordered. The manufacturer produces to satisfy the retailer’s order in full. The short production lead time enables the retailer to wait and improve her forecast before finalizing her ordering decision. The manufacturer has the capability to produce at a cheaper cost if the time pressure to build products is low. In other words, the manufacturer can build at a cheaper cost prior to receiving final orders from the retailer. We characterize this manufacturer’s optimal advance production quantity and the wholesale price. We show that the manufacturer optimally produces in two batches. We characterize the optimal quantity for the first batch that is built to stock prior to receiving an order from the retailer. After obtaining the forecast update, the retailer places an order and the manufacturer produces the second batch if needed. We also show that the manufacturer charges a higher wholesale price when the product’s market potential is high and when production is costly.

¹ Fraiman, N, Singh, M., 2002. Zara. Teaching case, Columbia Business School.

Second, we study the *dual purchase* contract that induces the retailer to place an order *before* observing the forecast update and an additional order *after* the forecast update. We show that a discount for advance orders; i.e., $w_a < w$, induces the retailer to place an order prior to obtaining the forecast update (or before the trade-show). In particular, the retailer optimally follows an *order-up-to policy*. In addition, we show that a lower advance purchase price or a higher wholesale price induces the retailer to place a larger order before the forecast update and a smaller order after the forecast update. Given the retailer's best response, we characterize the manufacturer's optimal advance production policy. We also show how a dual purchase contract enables the manufacturer to push inventory to the retailer and increase his expected profit.

Third, we investigate scenarios through which the manufacturer sets the dual purchase contract parameters. In the first scenario, the manufacturer sets only the advance purchase price while the wholesale price is exogenous. For example, the global computer memory prices for DRAMs are set by a spot market. Large retailers and DRAM manufacturer agree on a wholesale price based on the prevailing market price (Billington, 2002). Note, however, that the manufacturer can decide whether to provide a discount and the size of the discount for an advance purchase at his own discretion. In the present paper, we establish the manufacturer's optimal contract pricing strategy. We also characterize the conditions under which the manufacturer creates strict Pareto improvement over the wholesale price contract by using the dual purchase contract. In the second scenario, the manufacturer sets the wholesale price in addition to the advance purchase price. This case is observed when the manufacturer is the dominant party who dictates the contract terms. For example, PS2 game consoles are manufactured by Sony and the wholesale price is set exclusively by Sony.² For this case, we show that the manufacturer always prefers the dual purchase contract over the wholesale price contract. Our analytical results together with a numerical study show that market uncertainty is a key factor that defines when the dual purchase contract provides strict Pareto improvement over the wholesale price contract.

Next we show that the dual purchase contract improves a manufacturer's profit even when he does not have advance production capability. Note that when the manufacturer cannot produce at a cheaper cost for early orders, he has no reason to (hence he does not) build any inventory to stock. Surprisingly, the dual purchase contract improves even a *build-to-order* manufacturer's profit.

Finally, we study the impact of the dual purchase contract on a risk averse manufacturer. With a wholesale price contract, the manufacturer's profit is uncertain before the retailer commits to purchase any quantity. Reducing the resulting profit volatility is often more important than increasing the expected profit for a manufacturer when the manufacturer cannot diversify his financial risk. This is often the case when he invests in specialized equipment to build a single product or when he relies heavily on a single retailer to sell his product. We show that the manufacturer's profit volatility can be lowered with a dual purchase contract. We characterize the threshold risk aversion level above which any risk averse manufacturer would prefer a dual purchase contract over a wholesale price contract. We also show that the optimal advance purchase discount increases with the manufacturer's risk aversion factor.

Our results determine when a dual purchase contract, a simple price only contract, creates strict Pareto improvement over a wholesale price contract. Given the number of suppliers, customers, and products a firm has to manage, price-only contracts will continue to be the most common contracts in practice because of their simplicity. Hence, the dual purchase contract is a simple yet a powerful mechanism that increases profits while still being amenable to real applications.

2. Literature review

Supply chain literature studying the interaction between two firms often focuses on channel coordinating contracts, such as buy-back contracts, quantity flexibility contracts and revenue sharing contracts (Cachon, 2003). All these contracts involve terms other than the price. As Arrow (1985) and Lariviere and Porteus (2001) point out, such contracts incur administrative costs that are not explicitly included in their corresponding models. Many practitioners also point out the difficulties associated with administering complex contracts

² Kleindorfer and Wu, 2003 provides several other examples of custom and commodity products from various industries. A manufacturer of a highly specialized or customized product is likely to set the wholesale price.

(Billington and Kuper, 2003). Hence, we focus on a price-only contract, the dual purchase contract, because of its implementability and study whether it achieves strict Pareto improvement over a wholesale price contract when the supply chain obtains a forecast update. To do so, in Section 4 we extend the results of Lariviere and Porteus (2001) to account for a forecast update and advance production capability at the manufacturer. Next in Section 5, we fully develop the dual purchase contract and characterize the manufacturer's and retailer's optimal decisions.

A number of papers examine the impact of order timing on profit improvements in a supply chain established by a wholesale price contract. Cachon (2004) addresses inventory risk sharing with a newsvendor model. The retailer can order after the demand realization (in which case the manufacturer faces inventory risk) or before demand realization (in which case the retailer faces inventory risk). Iyer and Bergen (1997) and Ferguson et al. (2005) consider the impact of forecast updating on the order timing. Taylor (2006) addresses similar issues without a forecast update. However, unlike the above papers, the retailer sets the selling price. The author also investigates the affect of retailer sales effort and information asymmetry. None of the above authors consider the possibility of sequential decisions, two production modes, and the impact of forecast on production and ordering decisions *both* before and after the forecast update. Donohue (2000) considers two production modes and focuses on return option to achieve channel coordination. The present papers focus is on price-only contracts, the optimal prices, and strict Pareto improvement over the wholesale price contract.

Another stream of literature focuses on forecast information asymmetry. Cachon and Lariviere (2001) and Özer and Wei (2006) structure contracts that enable credible forecast information sharing between a manufacturer and a retailer. Note, however, that the manufacturer in the present paper does not need to observe the forecast update for his decisions. Hence, the firms in the present paper do not face an incentive problem due to forecast update information. For a discussion on asymmetric information models in supply chains, we refer the reader to Chen (2003). A final group of researchers study the effect of advance ordering where supply chain coordination is not an issue; i.e., either the retailer or the manufacturer is the only decision maker (Wheng and Parlar, 1999; Brown and Lee, 1998; Gallego and Özer, 2001; Tang et al., 2003; Erhun et al., 2003).

None of the above papers consider the effect of risk aversion. Eeckhoudt et al. (1995) study the comparative static of changes in price and cost parameters for a single risk averse newsvendor. Chen and Federgruen (2001) conduct a mean-variance analysis of basic inventory models and extend this analysis to include infinite horizon inventory models. Neither Eeckhoudt et al. nor Chen and Federgruen consider the effect of risk aversion in the supply chain context. We study the effect of risk aversion in the supply chain context, and show the value of a dual purchase contract for a risk averse manufacturer.

We organize the rest of the paper as follows. In Section 3, we present the demand model. In Section 4, we study the wholesale price contract and characterize the retailer's optimal ordering policy, the manufacturer's optimal production policy and his optimal wholesale price. In Section 5, we characterize optimal policies under the dual purchase contract. In Section 6, we show why the dual purchase contract improves supply chain efficiency. In Sections 7 and 8, we characterize the effect of a dual purchase contract when the manufacturer has only one production mode and when he is risk averse, respectively. In Section 9, we provide additional managerial insights through a numerical study. In Section 10, we conclude with possible future research directions.

3. The demand model

Consider a supply chain with a manufacturer and a retailer. The retailer buys a product from the manufacturer prior to a sales season. The manufacturer produces and satisfies all orders placed by the retailer in exchange for a payment based on the contract terms. Next, the market demand D is realized and the retailer satisfies as much customer demand as possible from available inventory on hand. The retailer's sales price is a fixed unit price $r > 0$.

Demand is of the form $D = X + \epsilon$ where X and ϵ are both uncertain. Before the selling season starts, the retailer learns X , which can be interpreted as a forecast update and is possibly obtained after a major trade show or market research. Prior to this market research, X is a continuous random variable with a cdf and a pdf of $F(\cdot)$ and $f(\cdot)$, respectively. We assume that the support of $f(\cdot)$ is $[\mu_L, \mu_H]$ where $\mu_H > \mu_L \geq 0$. We

Table 1
Glossary of notation

Cost parameters
 c_a : per unit production cost before the forecast update is realized
 c : per unit production cost after the forecast update is realized
 w_a : per unit advance purchase price
 w : per unit wholesale price
 r : per unit retail price

Uncertainty related notation
 X : random variable representing the forecast update
 μ : realization of forecast update X
 μ_L, μ_H : lowest and highest support of X
 $F(\cdot), f(\cdot)$: cdf and pdf of X
 ϵ : random variable representing the market uncertainty
 $G(\cdot), g(\cdot)$: cdf and pdf of ϵ

Decision variables
 x : retailer's advance order quantity
 x^* : retailer's optimal advance order quantity
 y : retailer's total order quantity
 $y^*(\mu)$: retailer's optimal total order quantity given the forecast update
 $y^{cs}(\mu)$: centralized system's optimal total order quantity
 z : manufacturer's advance production quantity
 z^* : manufacturer's optimal advance production quantity
 z^{cs} : centralized system's optimal advance production quantity
 w^* : manufacturer's optimal wholesale price
 w_a^* : manufacturer's optimal advance purchase price

use μ to denote the realization of X . The variable ϵ represents the residual market uncertainty and is realized after the sales season. We model ϵ as a continuous, mean-zero random variable. Hence, the mean demand before obtaining the forecast update is $\bar{\mu} \equiv EX$. We denote the cdf and pdf of ϵ with $G(\cdot)$ and $g(\cdot)$, respectively. The support of $g(\cdot)$ is $[a, b)$ where $-\mu_L \leq a < b \leq \infty$. This support ensures nonnegative demand. We assume that $G(\cdot)$ has an increasing failure rate (IFR). Distributions such as normal, gamma, and exponential have IFRs. For an easy reference, Table 1 summarizes the notation used throughout the paper.

The contractual agreement between the firms governs their actions and the resulting profits. We start our analysis with the wholesale price contract.

4. Wholesale price contract

The sequence of events under the wholesale price contract is summarized in Fig. 1. (1) Parties agree on a wholesale price w and sign the contract. Here, we do not assume a particular process by which the parties set the wholesale price. We will study possible processes later in Section 4.3. (2) The manufacturer decides how much to produce in advance at a per unit cost c_a , i.e., before receiving the retailer's order. (3) The retailer obtains the forecast update μ , which is the realization of X , and decides how much to order from the manufacturer. (4) The manufacturer produces an additional batch at a per unit cost c , if necessary, and satisfies the retailers order. At this stage production lead time requirement is short because the production is initiated

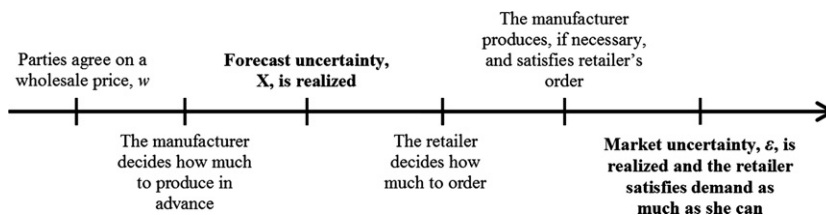


Fig. 1. Sequence of events under the wholesale price contract.

closer to the sales season. Hence, the manufacturer per unit production cost is higher than his cost in the earlier production stage; i.e., $c_a < c$. Finally, (5) market uncertainty ϵ is realized and the retailer satisfies demand from on-hand inventory at a fixed price r . We assume that $w \in [c, r]$; otherwise it is never profitable for the manufacturer to produce or the retailer to place any order.

Note that three decisions are made in a sequel: the wholesale price, the manufacturer’s advance production quantity and the retailer’s order quantity. We characterize the optimal decisions by using a backward induction algorithm; i.e., solve for the last decision first. The manufacturer’s production decision does not affect the retailer’s ordering policy. Hence, the retailer’s order decision depends only on the wholesale price. Therefore, first we solve the retailer’s ordering decision for a given wholesale price. The manufacturer’s advance production decision depends closely on the retailer’s order decision and the wholesale price. Hence, next we solve the manufacturer’s production problem. The pricing decision affects all other decisions. Therefore, we solve for the optimal wholesale price last.

4.1. The retailer’s problem

Given an order quantity y and the forecast update μ , the retailer’s expected profit is

$$\Pi^r(y, \mu) = rE_\epsilon[\min(y, \mu + \epsilon)] - wy. \tag{1}$$

The retailer maximizes this newsvendor problem and determines her optimal order quantity

$$y^*(\mu) \equiv \mu + G^{-1}\left(\frac{r-w}{r}\right). \tag{2}$$

4.2. The manufacturer’s problem

The manufacturer can save from production cost by initiating part of his production before the forecast update. At the time of the advance production decision, however, the retailer’s total order is unknown. Hence, the manufacturer trades off between excess inventory and lower production cost. Let $z \geq 0$ units be the manufacturer’s advance production quantity. Since the retailer orders only after the forecast update, the manufacturer builds all z units to stock. Given the advance production quantity and the retailer’s optimal order quantity, manufacturer’s expected profit under a wholesale price contract is

$$\Pi^m(w, z) \equiv wE_X y^*(X) - c_a z - cE_X(y^*(X) - z)^+. \tag{3}$$

Next we characterize the manufacturer’s advance production policy. We defer all the proofs and some of the technical lemmas to [Appendix A](#).

Theorem 1

1. The manufacturer optimally produces $z^* = F^{-1}\left(\frac{c-c_a}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)$ units in advance.
2. z^* is decreasing in c_a and $z^* \in (y^*(\mu_L), y^*(\mu_H))$.

This theorem characterizes the manufacturer’s *advance* production policy. Part 1 characterizes the optimal advance production quantity. From Eq. (2), the manufacturer knows that the retailer will order at least $y^*(\mu_L)$ units. Part 2 shows that the optimal production quantity is increasing in the cost saving from advance production. When the advance production cost is lower than c , the manufacturer optimally produces more than the minimum possible retailer order. Hence, he faces excess inventory risk. In particular, when $c_a < c$, the manufacturer optimally stocks more than $y^*(\mu_L)$.

4.3. Contract pricing decision

So far we have characterized the retailer’s optimal order size and the manufacturer’s optimal advance production quantity for a given wholesale price w . This scenario is possible when the product is a commodity and the firms take the wholesale price as given. Next, we study the scenario in which the manufacturer sets the

wholesale price as a Stackelberg leader. The retailer responds by setting her order size. This scenario is possible, for example, when the manufacturer builds a custom product and is the only producer of this product.

To solve for the manufacturer's optimal wholesale price, we substitute the optimal advance production quantity z^* characterized in Theorem 1 Part 1 to Eq. (3) and solve for

$$w^* \equiv \arg \max_w \Pi^m(w, z^*).$$

The next theorem characterizes the manufacturer's optimal wholesale price.

Theorem 2

1. The manufacturer's expected profit in Eq. (3) is unimodal in w . Hence, the manufacturer's optimal wholesale price is

$$w^* = r(1 - G(v^*)), \text{ where } v^* \text{ is the solution to}$$

$$r(1 - G(v)) \left(1 - \frac{g(v)(\bar{\mu} + v)}{1 - G(v)} \right) - c_a = 0.$$

2. w^* is increasing³ in $\bar{\mu}$.
3. w^* is increasing in c_a .

Part 1 characterizes the manufacturer's optimal wholesale price. Part 2 shows that the manufacturer's optimal wholesale price is higher when expected demand is high. Part 3 reveals that reducing the advance production cost enables the manufacturer to reduce his optimal wholesale price. These results show that the manufacturer would be charging a higher wholesale price when the market potential for the product is high and when it is costly to produce the product. The manufacturer, for example, can use a new technology or a process such as quick response initiatives to reduce the cost of production. The above theorem shows that the benefit of such improvements would be shared with the retailer through reduced wholesale prices.

5. Dual purchase contract

This contract specifies two prices, an advance purchase price w_a and the regular wholesale price w . The retailer pays w_a for each unit that she orders before observing the forecast update and w after the forecast update. The sequence of events under the dual purchase contract is summarized in Fig. 2. (1) Parties agree on w and w_a . Here, we do not assume a particular process by which contract parameters are set. Later, in Section 5.3, we address how the contract terms are set. (2) The retailer decides how much to order in advance of obtaining the forecast update and pays w_a per unit of advance order. (3) The manufacturer decides how much to produce in advance at a per unit production cost c_a . (4) The retailer obtains the forecast update μ and decides how much more to order and pays w per unit. (5) The manufacturer produces an additional batch at a per unit price c , if necessary, and satisfies the retailer's total order. (6) Market uncertainty ϵ is realized and the retailer satisfies demand from on-hand inventory as much as possible at a per unit price r .

Note that under this contract four decisions are made in a sequel: the dual purchase contract prices, the retailer's advance purchase quantity, the manufacturer's advance production quantity, and the retailer's regular order quantity. We solve for the optimal decisions by using a backward induction algorithm. The manufacturer's production decision does not affect the retailer's order quantity. Hence, first we solve for the retailer's problem. The manufacturer's advance production decision depends closely on the retailer's order decision. Hence, second we solve the manufacturer's problem. The pricing decision affects all other decisions. Hence, we solve for the contract-pricing problem last.

³ We use the terms "increasing" and "decreasing" in the strong sense; i.e., increasing means strictly increasing.

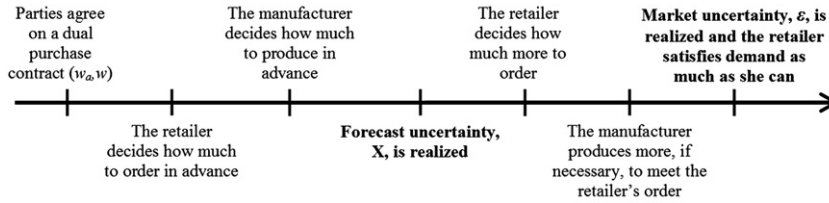


Fig. 2. Sequence of events under the dual purchase contract.

5.1. The retailer's problem

Given a dual purchase contract, the retailer has two opportunities to place orders: before and after the forecast update. While the retailer saves on procurement cost by placing an advance order, she obtains better information by placing an order after the forecast update. Thus, the retailer faces a tradeoff between lower cost and better information. Let x be the retailer's order quantity before the forecast update and y be the retailer's total order quantity after the forecast update. To maximize her profit, the retailer solves the following dynamic optimization problem:

$$\max_{x \geq 0} \Pi^r(x) = E_X \pi^r(x, X) - w_a x, \tag{4}$$

$$\text{where } \pi^r(x, \mu) = \max_{y \geq x} r E_\epsilon [\min(y, \mu + \epsilon)] - w(y - x). \tag{5}$$

When $w_a > w$, the optimal advance purchase quantity is zero because the retailer can purchase at a cheaper price after obtaining the forecast update. This case is equivalent to the wholesale price contract. Hence, in this section we consider $w_a \leq w$. The following theorem summarizes the retailer's optimal policy.

Theorem 3

1. The solution to Eq. (5) is $\max(y^*(\mu), x)$, where $y^*(\mu) \equiv \mu + G^{-1}(\frac{r-w}{r})$, and it is decreasing in w .
2. $\Pi^r(x)$ is concave in x and $\lim_{|x| \rightarrow \infty} \Pi^r(x) = -\infty$. Hence, the objective function in Eq. (4) has a maximizer x^* , which is the retailer's optimal advance purchase quantity.

This theorem shows that the retailer's optimal order policy is an *order-up-to* policy. Before observing the forecast update, the retailer orders up to x^* units. After the forecast update, if $x^* \leq y^*(\mu)$, she orders up to $y^*(\mu)$. Therefore, the retailer's additional order quantity after the forecast update is $(y^*(\mu) - x^*)^+$. Notice that order-up-to level $y^*(\mu)$ is the same as the optimal order quantity under the wholesale price contract given in Eq. (2). Hence, when the manufacturer offers a dual purchase contract, the retailer orders at least as much as what she would order under a wholesale price contract. This result implies that the dual purchase contract is a mechanism that induces the retailer to place additional orders. Next we further characterize the retailer's advance purchase quantity.

Theorem 4

1. When $w_a \in [c_a, \tau(w)]$ where $\tau(w) \equiv r \left[1 - \int_{\mu_L}^{\mu_H} G(y^*(\mu_H) - \mu) f(\mu) d\mu \right] < w$, the retailer orders only before the forecast update. In this case, x^* is the solution to the first order condition:

$$r \left[1 - \int_{\mu_L}^{\mu_H} G(x - \mu) f(\mu) d\mu \right] - w_a = 0. \tag{6}$$

x^* is decreasing in w_a and $x^* \geq y^*(\mu_H)$. When $w_a = \tau(w)$, we have $x^* = y^*(\mu_H)$.

2. When $w_a \in (\tau(w), w)$, the optimal advance purchase quantity x^* is the solution to the following first order condition:

$$w\left(1 - F\left(x - G^{-1}\left(\frac{r-w}{r}\right)\right)\right) + r\left[\int_{\mu_L}^{x-G^{-1}\left(\frac{r-w}{r}\right)} (1 - G(x - \mu))f(\mu) d\mu\right] - w_a = 0. \tag{7}$$

x^* is decreasing in w_a , increasing in w and $x^* \in (y^*(\mu_L), y^*(\mu_H))$.

3. When $w_a = w$, x^* can take any value between $[0, y^*(\mu_L)]$. The retailer's total purchase quantity and her expected profit under the dual purchase contract and under the wholesale price contract are equal. Hence, without loss of generality, $x^* = 0$.
4. $x^* > y^*(\mu_L)$ if and only if $w_a < w$.

Part 1 shows that when the advance purchase price w_a is below the threshold $\tau(w)$, the advance purchase quantity is larger than $y^*(\mu_H)$. This result, together with Theorem 3 Part 1, implies the following. The retailer orders only before observing the forecast update. Intuitively, the discount is deep enough to offset the retailer's gain from waiting and observing the forecast update before placing his order. Part 2 shows that when the advance purchase price $w_a \in (\tau(w), w)$, the optimal advance purchase quantity $x^* \in (y^*(\mu_L), y^*(\mu_H))$. Thus, given a discount, the retailer orders more than $y^*(\mu_L)$ units in advance and exactly $(y^*(\mu) - x^*)^+$ units after observing the forecast update. A higher wholesale price induces a larger advance purchase quantity. Part 3 shows that when $w_a = w$, the retailer orders nothing in advance. Part 4 states that a discount induces an advance purchase of $x^* > y^*(\mu_L)$.

5.2. The manufacturer's problem

Given the retailer's optimal response as characterized by Theorems 3 and 4, the retailer places part of her order before the forecast update. Hence, the manufacturer's expected profit under a dual purchase contract can be written as

$$\Pi_{dp}^m(z) = \begin{cases} w_a x^* - c_a z - c(x^* - z)^+, & w_a \in [c_a, \tau(w)], \\ w_a x^* + w E_X(y^*(X) - x^*)^+ - c_a z - c E_X(\max\{x^*, y^*(X)\} - z)^+, & w_a \in (\tau(w), w]. \end{cases} \tag{8}$$

To decide on the advance production quantity, the manufacturer solves

$$\max_{z \geq 0} \Pi_{dp}^m(z).$$

Next we characterize the advance production quantity z_{dp}^* .

Theorem 5

1. The profit function $\Pi_{dp}^m(z)$ is strictly concave in z .
2. When $w_a \in [c_a, \tau(w)]$, the optimal advance production quantity is $z_{dp}^* = x^*$. When $w_a \in (\tau(w), w]$, we have $z_{dp}^* = \max\{x^*, F^{-1}\left(\frac{c-c_a}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)\}$. For all w_a , we have $z_{dp}^* \geq F^{-1}\left(\frac{c-c_a}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)$.

This theorem fully characterizes the manufacturer's production policy. Part 2 shows that the manufacturer's optimal advance production is at least as much as the retailer's advance order quantity. This makes intuitive sense because the manufacturer's advance production cost is $c_a < c$. Part 2 also characterizes the manufacturer's minimum advance production quantity; i.e., he optimally produces at least $F^{-1}\left(\frac{c-c_a}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)$ units in advance.

5.3. Contract pricing decision

Here we solve for the optimal contract parameters w_a and w . We consider two possible scenarios. In the first scenario, the manufacturer and the retailer take the wholesale price w as given. However, the manufacturer decides whether to offer the advance purchase price w_a . If he does, he maximizes his profit by deciding on w_a . In most supply chains, the firms establish a relationship, hence a supply chain, for the product by agreeing

on a wholesale price long before the actual production is initiated or any forecast update is obtained. This price could also be set by the market. Neither the manufacturer nor the retailer may be able to enforce the wholesale price that maximizes their own profit. However, the manufacturer can offer an advance purchase price on his own discretion. In the second scenario, the manufacturer as the Stackelberg leader sets both w_a and w .

To obtain the optimal advance purchase price, the manufacturer solves the following problem:

$$w_a^* \equiv \operatorname{argmax}_{w_a \in [c_a, w]} \Pi_{dp}^m(w_a, z_{dp}^*), \tag{9}$$

where Π_{dp}^m is defined in Eq. (8). Next we characterize the solution.

Theorem 6. Let $w_\ell \equiv r(1 - G(v_\ell))$ where v_ℓ satisfies $r(1 - G(v_\ell)) \left[1 - \frac{g(v_\ell)(\mu_\ell + v_\ell)}{1 - G(v_\ell)} \right] = 0$.

1. When $w \leq w_\ell$, the optimal advance purchase price is $w_a^* = w$.
2. When $w > w_\ell$, the optimal advance purchase price is $w_a^* < w$.

The first part states that if the wholesale price is low; i.e., $w < w_\ell$, then the manufacturer should not offer an advance purchase discount and hence revert back to the wholesale price contract (simply by setting $w_a = w$). In this case, the retailer has no incentive to order in advance (recall from Theorem 4 Part 3). However, when the wholesale price is larger than the threshold w_ℓ , it is optimal for the manufacturer to provide a discount for orders placed prior to the forecast update.

Theorem 7. The dual purchase contract with (w_a^*, w) increases the manufacturer’s profit over the wholesale price contract if and only if $w > w_\ell$.

The retailer always has the option not to place any order before the forecast update. Therefore, the retailer is always better off with a dual purchase contract over a wholesale price contract when $w_a < w$. Hence, this theorem states that the manufacturer can strictly improve his expected profit as well as the retailer’s expected profit when he sets and offers the advance purchase price.

Now consider the scenario in which the manufacturer sets both w_a and w . To do so, he solves

$$\Pi_{dp}^* \equiv \max_{w_a, w} [\Pi_{dp}^m(w_a, w, z_{dp}^*)], \tag{10}$$

where $\Pi_{dp}^m(w_a, w, z_{dp}^*)$ is as defined in Eq. (8). We have the following result.

Theorem 8. When the manufacturer sets the wholesale price, the optimal dual purchase contract always increases his profit over the optimal wholesale price contract.

Hence, the manufacturer always prefers the dual purchase contract when he sets both prices. Whether the dual purchase contract also improves retailer’s profit over the wholesale price contract is not analytically conclusive when the manufacturer sets both prices. However, in most of our numerical experiments in Section 9, the retailer’s profit was also higher under dual purchase contract.

In brief, Theorem 7 states that when the manufacturer sets the advance purchase price, the dual purchase contract improves both the manufacturer’s and the retailer’s profits over the wholesale price contract, hence enabling strict Pareto improvement. Theorem 8 states that when the manufacturer sets the advance purchase price and the wholesale price, the dual purchase contract always improves the manufacturer’s profit over the optimal wholesale price contract.

6. Supply chain efficiency

This section provides the reason why the manufacturer can achieve higher profits with the dual purchase contract. To do so, we consider the centralized supply chain, for which no internal payment needs to be exchanged. In particular, we compare the resulting optimal profit of the centralized supply chain with the total supply chain profit of the decentralized system; i.e., the sum of the manufacturer’s and the retailer’s optimal

expected profits. When the centralized supply chain profit and decentralized total profit are equal, we say the system is coordinated. The difference in profits is a measure of decentralized supply chain’s efficiency.

The centralized system determines first the advance production quantity and the additional production quantity after obtaining the forecast update. Let z be the centralized system’s advance production quantity and y be the total production quantity after the forecast update. The centralized system solves the following dynamic program:

$$\max_{z \geq 0} \Pi^{cs}(z) = E_X \pi^{cs}(z, X) - c_a z, \tag{11}$$

$$\text{where } \pi^{cs}(z, \mu) = \max_{y \geq z} r E_c [\min(y, \mu + \epsilon)] - c(y - z). \tag{12}$$

The following theorem characterizes the centralized system’s production policy.

Theorem 9

1. The solution to (12) is $\max(z, y^{cs}(\mu))$ where $y^{cs}(\mu) = \mu + G^{-1}\left(\frac{r-\epsilon}{r}\right)$ for all $\mu \in [\mu_L, \mu_H]$.
2. $\Pi^{cs}(z)$ is concave in z and $\lim_{|z| \rightarrow \infty} \Pi^{cs}(z) = -\infty$. Hence, the objective function in Eq. (11) has a maximizer z^{cs} , which is the centralized system’s optimal advance production quantity.

This theorem characterizes the centralized system’s production policy to be a *produce-up-to* policy. The centralized system produces z^{cs} before the forecast update. After the forecast update, if $z^{cs} \leq y^{cs}(\mu)$, the centralized system produces up to $y^{cs}(\mu)$. Next we characterize z^{cs} .

Theorem 10

1. When $c_a \in [0, \tau^{cs}(c)]$ where $\tau^{cs}(c) \equiv r \left[1 - \int_{\mu_L}^{\mu_H} G(y^{cs}(\mu_H) - \mu) f(\mu) d\mu \right] < c$, the centralized system produces only before the forecast update. In this case, z^{cs} is the solution to the first order condition:

$$r \left[1 - \int_{\mu_L}^{\mu_H} G(z - \mu) f(\mu) d\mu \right] - c_a = 0. \tag{13}$$

z^{cs} is decreasing in c_a and $z^{cs} \geq y^{cs}(\mu_H)$. When $c_a = \tau^{cs}(c)$, we have $z^{cs} = y^{cs}(\mu_H)$.

2. When $c_a \in (\tau^{cs}(c), c)$, optimal advance production quantity z^{cs} is the solution to the following first order condition:

$$c \left(1 - F \left(z - G^{-1} \left(\frac{r-c}{r} \right) \right) \right) + r \left[\int_{\mu_L}^{z - G^{-1} \left(\frac{r-c}{r} \right)} (1 - G(z - \mu)) f(\mu) d\mu \right] - c_a = 0. \tag{14}$$

z^{cs} is decreasing in c_a , increasing in c and $z^{cs} \in (y^{cs}(\mu_L), y^{cs}(\mu_H))$.

3. When $c_a = c$, z^{cs} can take any value between $[0, y^{cs}(\mu_L)]$. Hence, without loss of generality, $z^{cs} = 0$.
4. $z^{cs} > y^{cs}(\mu_L)$ if and only if $c_a < c$.
5. $z^{cs} > F^{-1} \left(\frac{c-c_a}{c} \right) + G^{-1} \left(\frac{r-w}{r} \right)$ for all $c_a < c$.

The advance production and the total production quantity determine supply chain profit. Therefore, to achieve channel coordination in a decentralized system, the advance production quantity must be equal to z^{cs} and the total production quantity must be equal to $\max(z^{cs}, y^{cs}(\mu))$ for all $\mu \in [\mu_L, \mu_H]$.

First consider the wholesale price contract. It is well known that the wholesale price contract does not coordinate even the supply chain without a forecast update, resulting in supply chain inefficiency (Pasternack, 1985). This contract also does not coordinate the supply chain discussed here. To observe this, note that the manufacturer’s advance production quantity under this contract is less than the centralized system’s advance production quantity (compare Theorem 1 Part 1 with Theorem 10 Part 5). Next consider the dual purchase contract. Can a dual purchase contract coordinate, if not, improve the efficiency of the supply chain?

Theorem 11. *There always exists a dual purchase contract under which supply chain profit is greater than the profit under a given wholesale price contract. In other words, for any w , there exists w_a such that*

$E_X \Pi^r(y^*(X), X) + \Pi^m(w, z^*) < \Pi^r(x^*) + \Pi_{dp}^m(z_{dp}^*)$, where each profit function is defined in (1), (3), (4), and (8), respectively.

This result shows that a dual purchase contract mitigates the inefficiency caused by a wholesale price contract. The wholesale price contract causes double marginalization when $w > c$ (Pasternack, 1985). The dual purchase contract mitigates the adverse effect of double marginalization by providing an additional opportunity to order at a discounted advance purchase price $w_a < w$. Therefore, by offering an advance purchase price the manufacturer can increase the supply chain efficiency, retain part of the savings and leave the remaining to the retailer.

7. Without advance production

Next we investigate whether the strict Pareto improvement over the wholesale price contract is due to the advance production capability. In other words, can the manufacturer and the retailer be better off with a dual purchase contract than with a wholesale price contract even when $c_a > c$? To answer this question, first note that when $c_a > c$, the optimal advance production amount is zero under both contracts because the manufacturer can produce at a cheaper cost after obtaining retailer’s total order. Hence, the manufacturer produces in full after the retailer places her order. Note that the retailer’s order policy remains the same as before. She follows the optimal ordering policy characterized in the previous sections for both contracts. Hence, the only decision we need to analyze is the manufacturer’s pricing decision. We start with the wholesale price contract.

The manufacturer’s expected profit under the wholesale price contract can be written as

$$\Pi^m(w) = (w - c)E_X y^*(X). \tag{15}$$

The next theorem characterizes the manufacturer’s optimal.

Theorem 12

1. The manufacturer’s profit in Eq. (15) is unimodal in w . Hence, if he can, the manufacturer sets the wholesale price to

$$w^* = r(1 - G(v^*)), \text{ where } v^* \text{ is the solution to}$$

$$r(1 - G(v)) \left(1 - \frac{g(v)(\bar{\mu} + v)}{1 - G(v)} \right) - c = 0.$$

We denote the solutions of the above equations with w_L^* and v_L^* when the probability of facing the worst possible forecast realization is one ($\text{Prob}\{X = \mu_L\} = 1$), and hence $\bar{\mu} = \mu_L$.

2. w^* is increasing in $\bar{\mu}$ and $w^* \geq w_L^*$.
3. w^* is increasing in c .

From Part 1, if the manufacturer knows with certainty that the retailer will face the worst possible sales season; i.e., if $X = \mu_L$ with probability one, then he optimally sets the wholesale price equal to w_L^* . This wholesale price is the lowest price that the manufacturer would offer. Part 2 shows that the manufacturer optimally offers a higher wholesale price when the forecast update is expected to be high. Part 3 shows that the manufacturer optimally chooses a higher wholesale price if his regular production cost increases.

Without advance production, the manufacturer’s expected profit under the dual purchase contract simplifies to the following:

$$\Pi_{dp}^m(w_a) = \begin{cases} (w_a - c)x^*, & w_a \in [c, \tau(w)], \\ (w_a - c)x^* + (w - c)E_X(y^*(X) - x^*)^+, & w_a \in (\tau(w), w]. \end{cases} \tag{16}$$

Theorem 13. When $c_a > c$, the dual purchase contract with w_a^* increases the manufacturer’s profit over the wholesale price contract if and only if $w > w_L^*$.

This theorem shows that strict Pareto improvement under the dual purchase contract is achievable even for a manufacturer that does not have advance production capability. In particular, if the manufacturer can set

the advance purchase price, he can create a strict Pareto improvement over any wholesale price contract by offering the advance purchase price as long as $w > w_L^*$. The strict Pareto improvement in this case is also due to having two order opportunities. The discounted advance purchase price reduces the adverse affect of double marginalization.

8. Manufacturer’s risk attitude

So far, we have shown how a dual purchase contract increases expected profits. Our next aim is to characterize how the dual purchase contract also reduces the manufacturer’s profit volatility and how it affects a risk averse manufacturer.

Under a wholesale price contract, the manufacturer’s profit is uncertain prior to the forecast update. This uncertainty in profit projection often discourages capital investment. The manufacturer can, however, reduce this uncertainty by using a dual purchase contract.

Theorem 14. *Under a dual purchase contract where $c_a > c$, the variance of the manufacturer’s profit is $Var_{dp}(w_a) \equiv (w - c)^2 Var(y^*(X) - x^*)^+$, which is less than or equal to the variance of the manufacturer’s profit under the wholesale price contract. This variance equals zero when $w_a \in [c, \tau(w)]$ and it is increasing in w_a when $w_a \in (\tau(w), w]$.*

Note that the variance depends on w_a through the retailer’s optimal advance purchase quantity x^* . A dual purchase contract reduces the manufacturer’s profit volatility and, hence, enables risk hedging. If a manufacturer is risk averse, then he can determine the optimal advance purchase price by trading off expected profit and profit variance. The mean-variance tradeoff is widely used in portfolio theory and can be reconciled with the expected utility approach by using a quadratic utility function. Consider utility function $u(x) = ax - \frac{1}{2}bx^2$ ($a > 0, b \geq 0, x \leq a/b$). When x is random, let $\bar{x} \equiv E(x)$. The expected utility is $E[u(x)] = a\bar{x} - \frac{1}{2}b\bar{x}^2 - \frac{1}{2}bVar(x)$. Clearly, for all feasible x ’s with the same expected value, the optimal one must have minimum variance. Alternatively, note that maximizing expected utility is equivalent to maximizing the certainty equivalent, which can be approximated as $c \sim \bar{x} + \frac{1}{2} \frac{u''(\bar{x})}{u'(\bar{x})} Var(x)$ (Luenberger, 1998, p. 256). For utility functions with constant absolute risk aversion k , we have $\frac{u''(\bar{x})}{u'(\bar{x})} = -k$.

Specifically, the manufacturer solves the following optimization problem:

$$\max_{w_a} \Pi_{ra}^m(w_a) \equiv \Pi_{dp}^m(w_a) - kVar_{dp}(w_a). \tag{17}$$

Note that, given the retailer’s response, the manufacturer’s objective function is

$$\Pi_{ra}^m(w_a) = \begin{cases} (w_a - c)x^*, & w_a \in [c, \tau(w)], \\ (w_a - c)x^* + (w - c)E_X(y^*(X) - x^*)^+ - k(w - c)^2Var_X(y^*(X) - x^*)^+, & w_a \in (\tau(w), w]. \end{cases} \tag{18}$$

The coefficient k reflects the manufacturer’s risk attitude. When $k = 0$, the manufacturer is risk neutral as in the previous section. When $k > 0$, the manufacturer is risk averse and a larger k implies that the manufacturer is more risk averse. When $k < 0$, the manufacturer is risk seeking.

Theorem 15. *Let $\underline{k} \equiv -\frac{w-c-r\gamma^*(\mu_L)g(y^*(\mu_L)-\mu_L)}{2(w-c)^2(\bar{\mu}-\mu_L)}$.*

1. *When $k > \underline{k}$, the manufacturer prefers the dual purchase contract with (w_a^*, w) to a wholesale price contract with w . The optimal advance purchase price w_a^* is decreasing in k .*
2. *When $k \leq \underline{k}$, the manufacturer prefers the wholesale price contract with w to a dual purchase contract with (w_a, w) .*

The theorem states that when the manufacturer’s risk aversion factor is above the threshold \underline{k} , he always prefers a dual purchase contract. The theorem also characterizes the risk aversion threshold above which any manufacturer would prefer the dual purchase contract over the wholesale price contract. Furthermore,

to increase the advance purchase quantity and hence reduce his profit volatility, a manufacturer with higher risk aversion offers a greater discount.

9. Numerical examples

The purpose of this section is to illustrate some of our results and numerically compare the profits under the dual purchase contract to the profits under the wholesale price contract. Our base case is $c_a = 2.5$, $c = 3$ and $r = 10$. The forecast update X follows the truncated normal distribution on $[8, 24]$ with mean 16 and standard deviation σ_X . The market uncertainty ϵ also follows the truncated normal distribution on $[-8, 8]$ with mean 0 and standard deviation σ_ϵ . The truncated normal distribution has IFR.

Consider the pricing scenario (discussed in Section 5.3) in which the manufacturer sets the advance purchase price w_a while w is exogenously set by the market. We have shown that the manufacturer achieves strict Pareto improvement over the wholesale price contract by offering a dual purchase contract as long as $w > w_\ell$ (Theorem 7). Hence, the set of wholesale prices for which the dual purchase contract provides strict Pareto improvement is larger when w_ℓ is smaller. Table 2 provides the manufacturer’s and the retailer’s expected profits under various parameter settings. Both firms’ expected profits are higher under the dual purchase contract. Hence, the manufacturer creates a strict Pareto improvement or a mutually beneficial proposition by introducing the dual purchase contract. Table 2 also illustrates the sensitivity of w_ℓ with respect to the variability in the forecast update X and market uncertainty ϵ . In these experiments, we keep the supports and means of X and ϵ fixed while varying the value of σ_X and σ_ϵ . Note that w_ℓ decreases as σ_ϵ increases. Therefore, the dual purchase contract achieves strict Pareto improvement for a larger set of wholesale prices when the market uncertainty ϵ is more variable. Note also that w_ℓ depends on μ_L (Theorem 6) but not on σ_X . In summary, the market uncertainty plays a critical role in defining when the manufacturer should consider using a dual purchase contract instead of a wholesale price contract.

Table 2 also illustrates the percentage increase in total supply chain profit by switching to the dual purchase contract (see the columns titled as “%”). Notice that the percentage improvement is higher when the ratio σ_X/σ_ϵ is low. This ratio represents the informative value of the forecast update. A low σ_X/σ_ϵ value implies that most of the demand uncertainty remains to be resolved after the forecast update. Therefore, our experiments suggest that the dual purchase contract improves the supply chain more when the forecast update is not very informative. If the manufacturer knows that the trade show or the market research conducted to obtain a forecast update is likely to be not informative, then he should highly consider offering the dual purchase contract.

Consider next the pricing scenario in which the manufacturer sets both w_a and w . When the manufacturer offers the wholesale price contract, he sets w to maximize his profit in Eq. (3). His optimal wholesale price w^* is

Table 2
Pareto improvement when $w = 9, 8, 7$, and 6 and $c_a = 2.5$, $c = 3$, $r = 10$

σ_ϵ	σ_X/σ_ϵ	w_ℓ	Wholesale $w = 9$		Dual purchase ($w_a^*, 9$)			%	Wholesale $w = 8$		Dual purchase ($w_a^*, 8$)			%
			Π^m	Π^r	Π_{dp}^m	Π^r	w_a^*		Π^m	Π^r	Π_{dp}^m	Π^r	w_a^*	
3.50	1.14	6.48	70.43	9.99	71.54	12.68	8.81	4.73	67.68	22.11	67.87	24.19	7.88	2.53
4.00	1.00	5.84	65.21	9.36	66.69	13.43	8.72	7.44	64.01	20.73	64.59	24.36	7.80	4.97
4.50	0.89	5.16	60.40	8.88	62.23	15.04	8.58	11.53	59.69	19.46	60.62	23.03	7.83	5.69
3.50	1.29	6.48	70.13	9.99	71.29	12.84	8.80	5.00	67.39	22.11	67.66	24.99	7.83	3.52
4.00	1.13	5.84	64.92	9.36	66.52	14.13	8.66	8.58	63.72	20.73	64.38	24.02	7.82	4.68
4.50	1.00	5.16	60.12	8.88	62.15	15.80	8.50	12.97	59.41	19.47	60.34	22.90	7.84	5.53
			$w = 7$		$(w_a^*, 7)$			%	$w = 6$		$(w_a^*, 6)$			%
			Π^m	Π^r	Π_{dp}^m	Π^r	w_a^*		Π^m	Π^r	Π_{dp}^m	Π^r	w_a^*	
3.50	1.14	6.48	65.56	35.57	65.57	35.97	6.99	0.40	55.62	50.11	55.62	50.11	6.00	0.00
4.00	1.00	5.84	63.29	33.62	63.36	35.26	6.94	1.76	54.70	47.82	54.71	47.82	5.99	0.01
4.00	0.89	5.16	60.29	31.66	60.65	36.64	6.78	5.81	53.43	45.39	53.46	46.86	5.96	1.52
3.50	1.29	6.48	64.92	35.57	64.93	36.25	6.98	0.69	54.98	50.11	54.98	50.11	6.00	0.00
4.00	1.13	5.84	62.65	33.62	62.76	36.11	6.90	2.70	54.06	47.82	54.07	47.83	5.99	0.02
4.50	1.00	5.16	59.65	31.65	60.17	37.80	6.72	7.31	52.79	45.39	52.84	47.67	5.94	2.37

Table 3
Pareto improvement when manufacturer sets w

σ_X	σ_ϵ	σ_X/σ_ϵ	Wholesale			Dual purchase			%
			Π^m	Π^r	w^*	Π_{dp}^m	Π^r	$(w_a, w)^*$	
$c_a = 2, c = 3, r = 6$									
4.00	3.50	1.14	33.86	5.77	5.42	35.30	7.65	(5.29, 5.62)	8.38
4.00	4.00	1.00	31.14	6.04	5.36	33.03	7.73	(5.28, 5.58)	9.63
4.00	4.50	0.89	28.62	5.14	5.42	31.02	6.28	(5.40, 5.72)	10.49
4.50	3.50	1.29	33.22	5.77	5.42	34.94	7.72	(5.27, 5.62)	9.41
4.50	4.00	1.13	30.50	6.04	5.36	32.69	7.79	(5.26, 5.58)	10.78
4.50	4.50	1.00	27.98	5.14	5.42	30.69	6.31	(5.38, 5.72)	11.71
$c_a = 2.5, c = 3, r = 6$									
4.00	3.50	1.14	29.89	4.67	5.52	30.73	5.35	(5.48, 5.58)	4.40
4.00	4.00	1.00	27.49	4.79	5.48	28.44	5.55	(5.45, 5.54)	5.30
4.00	4.50	0.89	25.49	3.07	5.64	26.64	3.87	(5.61, 5.82)	6.83
4.50	3.50	1.29	29.60	4.67	5.52	30.42	5.41	(5.48, 5.56)	4.55
4.50	4.00	1.13	27.20	4.79	5.48	28.10	5.54	(5.45, 5.56)	5.16
4.50	4.50	1.00	25.20	3.07	5.64	26.40	3.86	(5.60, 5.88)	7.04
$c_a = 2, c = 3, r = 10$									
4.00	3.50	1.14	74.73	12.52	8.78	76.17	15.89	(8.56, 9.16)	5.51
4.00	4.00	1.00	69.51	13.93	8.58	71.71	17.19	(8.43, 8.98)	6.54
4.00	4.50	0.89	64.26	14.18	8.48	67.23	17.00	(8.43, 8.98)	7.38
4.50	3.50	1.29	74.09	12.52	8.78	75.97	15.72	(8.54, 9.18)	5.87
4.50	4.00	1.13	68.87	13.93	8.58	71.52	17.03	(8.42, 9.00)	6.94
4.50	4.50	1.00	63.62	14.18	8.48	67.05	16.82	(8.42, 9.00)	7.80
$c_a = 2.5, c = 3, r = 10$									
4.00	3.50	1.14	70.50	11.36	8.88	71.54	12.70	(8.81, 9.00)	2.91
4.00	4.00	1.00	65.51	12.37	8.72	66.73	14.20	(8.65, 8.88)	3.92
4.00	4.50	0.89	60.61	12.09	8.68	62.27	14.42	(8.64, 9.12)	5.49
4.50	3.50	1.29	70.20	11.36	8.88	71.29	12.86	(8.80, 9.00)	3.18
4.50	4.00	1.13	65.22	12.37	8.72	66.53	14.62	(8.62, 8.94)	4.59
4.50	4.50	1.00	60.31	12.09	8.68	62.22	14.38	(8.63, 9.24)	5.80

characterized in [Theorem 2](#) Part 1. When he offers the dual purchase contract, he sets both w_a and w to maximize his profit. He solves the optimization problem in Eq. (10). In [Table 3](#), we compare the profits resulting from the manufacturer's optimal choices. As [Theorem 8](#) proves, the manufacturer's profit is always higher under the dual purchase contract. In all the experiments, the retailer's profit is also higher under the dual purchase contract. The percentage improvement in supply chain profit tends to increase when σ_X/σ_ϵ is low. Comparing the $c_a = 2.5, c = 3$ and the $c_a = 2, c = 3$ cases, we note that the percentage improvement is higher when the cost difference between the advance production and normal production is high. Intuitively, the dual purchase contract allows the supply chain to enjoy low production costs by providing the retailer incentive to place most of her need through advance orders.

10. Conclusion

We study a supply chain in which the retailer observes a forecast update and the manufacturer produces to satisfy the retailer's order. Under a wholesale price contract, the retailer places orders only after observing the forecast update. Preserving the simplicity, we study another price-only contract, the dual purchase contract, which provides the retailer with incentive and flexibility to order both before and after the forecast update. We show that the dual purchase contract increases supply chain profit as well as the retailer's profit, and identify conditions under which it also increases the manufacturer's profit and hence creates a strict Pareto improvement. The analytical results together with the numerical examples suggest that the dual purchase contract is

likely to achieve strict Pareto improvement over the wholesale price contract when (1) low cost advance production is available, (2) the manufacturer is risk averse, (3) the market uncertainty is high, or (4) the worst possible forecast update value is low.

We have also investigated other supply chain scenarios (Wei, 2003). For example, results similar to those in Theorem 7 are shown when the manufacturer faces excess inventory risk due to an exogenous minimum production quantity requirement. Manufacturers must often keep labor force utilization at a production facility above a certain percentage due to long term labor contracts (see Fisher and Raman, 1996, for examples). The imposed minimum production quantity generates excess inventory risk similar to the case of advance production. Theorem 7 continues to hold in this case. Practitioners often characterize the forecast update X with simple distributions such as low and high forecast update with a Bernoulli distribution. For such cases, the thresholds and the decision variables, such as the optimal advance purchase price w_a^* are characterized in closed-form solutions.

Price-only contracts are the most prevalent contracts used in practice. Hence, additional research on such contracts is needed and will likely to have real impact in practice. There are several fertile avenues for future research. For example, here we studied per unit payments that are independent of the order size. An extension is to consider a quantity discount scheme. Quantity discounts can induce even larger orders from the retailer for situations when the forecast realization is low. Note, however, that quantity discount also reduces the manufacturer's profit. Based on the results in this paper, we conjecture that such a contract will also achieve Pareto improvement over the wholesale price contract. However, the net effect is not quantitatively apparent. Analyzing a dual purchase contract with a quantity discount scheme is an interesting future research question.

Appendix A. Proofs

A.1. Proofs of theorems in Section 4 and required technical lemmas

Proof of Theorem 1. To prove Part 1, we define $v \equiv G^{-1}\left(\frac{r-w}{r}\right)$ and rewrite the manufacturer's profit in Eq. (3) as

$$\Pi^m(w, z) = w(\bar{\mu} + v) - c_a z - c \int_{z-v}^{\mu_H} (\mu + v - z) f(\mu) d\mu. \tag{19}$$

Then, we have $\frac{d\Pi^m(w, z)}{dz} = -c_a + c(1 - F(z - v))$. Since $F(\cdot)$ is increasing on $[\mu_L, \mu_H]$, we conclude that $\Pi^m(w, z)$ is strictly concave in z on $[\mu_L + v, \mu_H + v]$ and z^* is the solution to the first order condition. Hence, $z^* = F^{-1}\left(\frac{c-c_a}{c}\right) + v$.

Part 2 follows from Part 1 because the forecast update, X , is distributed on $[\mu_L, \mu_H]$. \square

Proof of Theorem 2. First we provide the following lemma as an auxiliary proposition to simplify the proof of this theorem.

Lemma 1

Let $\lambda(v)$ be any differentiable function. If

$$\frac{d\lambda(v)}{dv} = \alpha \left[r(1 - G(v)) \left(1 - \frac{g(v)(\mu + v)}{1 - G(v)} \right) - \gamma(v) \right],$$

where $\alpha > 0$; $\mu \geq 0$; $r > \gamma(v) \geq 0$; $\gamma(v)$ nondecreasing, then $\lambda(v)$ is unimodal on R .

Proof of Lemma 1. Let \bar{v}_o be the supremum of the set of points such that $\frac{g(v)v}{1-G(v)} \leq 1$. First, we prove that \bar{v}_o is finite. Lariviere and Porteus (2001, Lemma 2(c)) proves that \bar{v}_o is finite when the lowest possible support for ϵ is $a = 0$. We use a similar argument as in their proof but consider the case where a can take a finite negative value. Let $t \in (0, b)$ and define the truncated random variable ϵ_t on $[t, b)$, whose cdf is $G_t(v) = \frac{G(v)-G(t)}{1-G(t)}$ for all $v \in [t, b)$. Note that ϵ_t has a finite mean since ϵ has a finite mean (in particular, zero mean). Note also that the failure rate of ϵ_t is equal to the failure rate of ϵ for all $v \in [t, b)$. Denote the failure rate of ϵ_t with $h_t(\cdot)$. Assume for a contradiction argument that $\bar{v}_o = \infty$. Then, $h_t(v) \leq \frac{1}{v}$ for all $v \in [t, b)$. This implies that ϵ_t is

stochastically larger than a random variable η with a failure rate $h_\eta(v) = \frac{1}{v}$ for $v \geq t$ (Ross, 1983). We have $1 - G_\eta(v) = \frac{1}{v}$ for all $v \geq t$ since $1 - G_\eta(v) = \exp[-\int_t^v h_\eta(v) dv]$ (Ross, 1983). Then, η has an infinite mean implying that ϵ_t has an infinite mean, too. This contradicts with the fact that ϵ_t has a finite mean. Therefore, \bar{v}_o must be finite. This result implies that \bar{v} , the supremum of the set of points such that $\frac{g(v)(\mu+v)}{1-G(v)} \leq 1$, is also finite because $\mu \geq 0$.

The second derivative of $\lambda(v)$ is

$$\frac{d^2\lambda(v)}{dv^2} = -\alpha r g(v) \left(1 - \frac{g(v)(\mu+v)}{1-G(v)}\right) - \alpha r (1-G(v)) \frac{d}{dv} \left(\frac{g(v)(\mu+v)}{1-G(v)}\right) - \alpha \frac{d\gamma(v)}{dv}.$$

For $v \in (-\infty, a)$, we have $\frac{d\lambda(v)}{dv} = r - \gamma(v) > 0$ and hence $\lambda(v)$ is increasing. Since $G(\cdot)$ has IFR, we have $\frac{d}{dv} \left(\frac{g(v)(\mu+v)}{1-G(v)}\right) > 0$ for all $v \in [a, b)$. Then for $v \in [a, \bar{v}]$, we have $\frac{d^2\lambda(v)}{dv^2} < 0$ and hence $\lambda(v)$ is strictly concave. For $v \in (\bar{v}, \infty)$, we have $\frac{d\lambda(v)}{dv} < 0$ and hence $\lambda(v)$ is decreasing. Therefore, $\lambda(v)$ is unimodal on R and its maximizer lies on $[a, \bar{v}]$. \square

Next we prove the theorem. To prove Part 1, we define $v \equiv G^{-1}\left(\frac{r-w}{r}\right)$. Then, we have $w = r(1 - G(v))$. Using this and substituting z^* from Theorem 1 Part 1 into Eq. (3) we rewrite the manufacturer’s profit as

$$\Pi^m(v, z^*) = (r(1 - G(v)) - c_a)(\bar{\mu} + v) + c_a \bar{\mu} - c \int_{F^{-1}\left(\frac{c-c_a}{c}\right)}^{\mu_H} \mu f(\mu) d\mu. \tag{20}$$

Hence, $\frac{d\Pi^m(v, z^*)}{dv} = r(1 - G(v)) \left(1 - \frac{g(v)(\bar{\mu}+v)}{1-G(v)}\right) - c_a$. From Lemma 1, $\Pi^m(v, z^*)$ is unimodal in v , and the optimal v is the solution to the first order condition as stated in the theorem. Since v is monotone in w , the profit function is also unimodal in w .

To prove Part 2, let $\Pi^m(v, z^*, \mu) \equiv (r(1 - G(v)) - c_a)(\mu + v) + c_a \mu - c \int_{F^{-1}\left(\frac{c-c_a}{c}\right)}^{\mu_H} \mu f(\mu) d\mu$, and v_1 and v_2 be the profit maximizing quantity for $\mu_1 < \mu_2$, respectively. For any $v > 0$, we have

$$\begin{aligned} \frac{\partial \Pi^m(v, z^*, \mu_2)}{\partial v} &= r(1 - G(v)) \left(1 - \frac{g(v)(\mu_2 + v)}{1-G(v)}\right) - c_a < r(1 - G(v)) \left(1 - \frac{g(v)(\mu_1 + v)}{1-G(v)}\right) - c_a \\ &= \frac{\partial \Pi^m(v, z^*, \mu_1)}{\partial v}. \end{aligned}$$

Hence, $\left. \frac{\partial \Pi^m(v, z^*, \mu_2)}{\partial v} \right|_{v=v_1} < \left. \frac{\partial \Pi^m(v, z^*, \mu_1)}{\partial v} \right|_{v=v_1} = 0$. Since $\Pi^m(v, z^*, \mu_2)$ is unimodal in v , we have $v_2 < v_1$, and hence $w^*(\mu_2) = r(1 - G(v_2)) > r(1 - G(v_1)) = w^*(\mu_1)$. Therefore, w^* is increasing in $\bar{\mu}$.

To prove Part 3, let $\Pi^m(v, z^*, c_a) \equiv [r(1 - G(v)) - c_a](\bar{\mu} + v) + c_a \bar{\mu} - c \int_{F^{-1}\left(\frac{c-c_a}{c}\right)}^{\mu_H} \mu f(\mu) d\mu$, and v_1 and v_2 be the profit maximizing quantity for $c_{a1} < c_{a2}$, respectively. We have

$$\begin{aligned} \frac{\partial \Pi^m(v, z^*, c_{a2})}{\partial v} &= r(1 - G(v)) \left(1 - \frac{g(v)(\bar{\mu} + v)}{1-G(v)}\right) - c_{a2} < r(1 - G(v)) \left(1 - \frac{g(v)(\bar{\mu} + v)}{1-G(v)}\right) - c_{a1} \\ &= \frac{\partial \Pi^m(v, z^*, c_{a1})}{\partial v}. \end{aligned}$$

Hence $\left. \frac{\partial \Pi^m(v, z^*, c_{a2})}{\partial v} \right|_{v=v_1} < \left. \frac{\partial \Pi^m(v, z^*, c_{a1})}{\partial v} \right|_{v=v_1} = 0$. Since $\Pi^m(v, z^*, c_{a2})$ is unimodal in v , we have $v_2 < v_1$, and hence $w^*(c_{a2}) = r(1 - G(v_2)) > r(1 - G(v_1)) = w^*(c_{a1})$. Therefore, w^* is increasing in c_a . \square

A.2. Proofs of theorems in Section 5 and required technical lemmas

Proof of Theorem 3. To prove Part 1, note that the objective function in Eq. (5) is concave in y because $\min(y, \mu + \epsilon)$ is concave and expectation preserves concavity. Therefore, its maximizer is $y^*(\mu) = \mu + G^{-1}\left(\frac{r-w}{r}\right)$, which is decreasing in w . The maximizer for the constrained problem is $\max(y^*(\mu), x)$.

To prove Part 2, note from Part 1 that we have

$$\pi^r(x, \mu) = \begin{cases} rE[\min(y^*(\mu), \mu + \epsilon)] - w(y^*(\mu) - x), & x < y^*(\mu), \\ rE[\min(x, \mu + \epsilon)], & x \geq y^*(\mu), \end{cases}$$

which is concave in x . Since expectation preserves concavity and the sum of two concave functions is concave, the objective function in Eq. (4) is also concave in x . We also have $\lim_{|x| \rightarrow \infty} \Pi'(x) = -\infty$. Hence, x^* , the maximizer of the objective function, is finite. \square

Proof of Theorem 4. To prove Parts 1 and 2, we define $v \equiv G(\frac{r-w}{r})$ and rewrite the objective function in Eq. (4) using Theorem 3 Parts 1 and 2:

$$\Pi'(x) = \begin{cases} rE \min(y^*(X), X + \epsilon) - wE(y^*(X) - x) - w_a x, & x < y^*(\mu_L), \\ \int_{\mu_L}^{x-v} rE \min(x, \mu + \epsilon) f(\mu) d\mu + \int_{x-v}^{\mu_H} [rE \min(y^*(\mu), \mu + \epsilon) - w(y^*(\mu) - x)] f(\mu) d\mu - w_a x, & x \in [y^*(\mu_L), y^*(\mu_H)] \\ rE \min(x, X + \epsilon) - w_a x, & x > y^*(\mu_H). \end{cases}$$

To prove Part 1, we first show that $\tau(w) < w$. Note that, $\tau(w) = r[1 - \int_{\mu_L}^{\mu_H} G(\mu_H + v - \mu) f(\mu) d\mu] < r[1 - \int_{\mu_L}^{\mu_H} \frac{r-w}{r} f(\mu) d\mu] = w$ because $G(\mu_H + v - \mu) > G(v) = \frac{r-w}{r}$ for all $\mu \in [\mu_L, \mu_H]$.

Next for $w_a \in [c_a, \tau(w)]$, we prove that x^* satisfies the first order condition in Eq. (6). Note from above that $w_a < w$ for all $w_a \in [c_a, \tau(w)]$. When $x < y^*(\mu_L)$, the objective function is increasing in x and maximized at $y^*(\mu_L)$ because $\frac{d\Pi'(x)}{dx} = w - w_a > 0$.

When $x \in [y^*(\mu_L), y^*(\mu_H)]$, we have

$$\frac{d\Pi'(x)}{dx} = w(1 - F(x - v)) + r \left[\int_{\mu_L}^{x-v} (1 - G(x - \mu)) f(\mu) d\mu \right] - w_a. \tag{21}$$

Notice that $\frac{d\Pi'(x)}{dx}$ is decreasing in x from $\frac{d\Pi'(x)}{dx} \Big|_{x=y^*(\mu_L)} = w - w_a > 0$ down to $\frac{d\Pi'(x)}{dx} \Big|_{x=y^*(\mu_H)} = \tau(w) - w_a \geq 0$.

Hence, in this region, the objective function is still increasing and hence, maximized at $y^*(\mu_H)$.

When $x > y^*_H$ we have

$$\frac{d\Pi'(x)}{dx} = r \left[1 - \int_{\mu_L}^{\mu_H} G(x - \mu) f(\mu) d\mu \right] - w_a. \tag{22}$$

Notice that $\frac{d\Pi'(x)}{dx}$ is decreasing in x from $\tau(w) - w_a \geq 0$. Hence, x that satisfies $\frac{d\Pi'(x)}{dx} = 0$ is larger than or equal to $y^*(\mu_H)$ and satisfies the first order condition in Eq. (6). These three cases imply that the optimal advance purchase quantity x^* satisfies Eq. (6). Note that when $w_a = \tau(w)$, we have $x^* = y^*(\mu_H)$.

To prove that x^* is decreasing in w_a , let $w_a^1 < w_a^2$ and x_1^*, x_2^* be the corresponding optimum advance purchase quantities. Then, by Eq. (6), we have $\int_{\mu_L}^{\mu_H} G(x_1^* - \mu) f(\mu) d\mu > \int_{\mu_L}^{\mu_H} G(x_2^* - \mu) f(\mu) d\mu$. This implies that $x_1^* > x_2^*$ and concludes the proof of Part 1.

To prove Part 2, for $w_a \in (\tau(w), w)$, we show that x^* satisfies the first order condition in Eq. (7). When $x < y^*(\mu_L)$, the objective function is increasing in x and maximized at $y^*(\mu_L)$ because $\frac{d\Pi'(x)}{dx} = w - w_a > 0$.

When $x \in [y^*(\mu_L), y^*(\mu_H)]$, $\frac{d\Pi'(x)}{dx}$, as defined in (21), is decreasing in x from $\frac{d\Pi'(x)}{dx} \Big|_{x=y^*(\mu_L)} = w - w_a > 0$ down to $\frac{d\Pi'(x)}{dx} \Big|_{x=y^*(\mu_H)} = \tau(w) - w_a < 0$. Hence, in this region, the maximizer satisfies the first order condition in Eq. (7).

When $x > y^*(\mu_H)$, the objective function is decreasing in x because $\frac{d\Pi'(x)}{dx}$, as defined in (22), is decreasing in x from $\tau(w) - w_a < 0$. These three cases imply that the optimal advance purchase quantity x^* satisfies Eq. (7).

To prove that x^* is decreasing in w_a , let $w_a^1 > w_a^2$ and x_1^*, x_2^* be the corresponding optimal advance purchase quantities. Note that Eq. (7) implies $x_1^* \neq x_2^*$. Now, assume for a contradiction argument that $x_1^* > x_2^*$. Then, from Eq. (7), we have

$$0 < w_a^1 - w_a^2 = r \int_{\mu_L}^{x_2^*-v} (G(x_2^* - \mu) - G(x_1^* - \mu)) f(\mu) d\mu + \int_{x_2^*-v}^{x_1^*-v} (r(1 - G(x_1^* - \mu)) - w) f(\mu) d\mu.$$

The first term above is nonpositive since $x_2^* < x_1^*$ from our assumption. To observe that second term is also nonpositive, note that for all $\mu < x_1^* - v$, we have $r(1 - G(x_1^* - \mu)) < w$. Therefore, the total is nonpositive which leads to a contradiction.

To prove that x^* is increasing in w , let $w_1 > w_2$ and x_1^*, x_2^* be the corresponding optimal advance purchase quantities. Define $v_1 \equiv G^{-1}\left(\frac{r-w_1}{r}\right)$, $v_2 \equiv G^{-1}\left(\frac{r-w_2}{r}\right)$ and note that $v_1 < v_2$. We hold w_a constant while changing w . Therefore, Eq. (7) implies that $x_1^* \neq x_2^*$. Now, assume for a contradiction argument that $x_1^* < x_2^*$. There are two cases to consider:

Case 1: $x_1^* - v_1 < x_2^* - v_2$. We find the value of w_a from Eq. (7) and check that it remains constant as we change the value of w . From Eq. (7), the difference between the value of w_a when $w = w_1$ and $w = w_2$ is

$$\int_{\mu_L}^{x_1^*-v_1} r(G(x_2^* - \mu) - G(x_1^* - \mu))f(\mu) d\mu + \int_{x_1^*-v_1}^{x_2^*-v_2} (w_1 - r(1 - G(x_2^* - \mu)))f(\mu) d\mu + \int_{x_2^*-v_2}^{\mu_H} (w_1 - w_2)f(\mu) d\mu.$$

The first term is nonnegative since $x_1^* < x_2^*$ from our assumption. The second term is also nonnegative because $G(\cdot)$ is increasing and $(w_1 - r(1 - G(x_2^* - \mu)))|_{\mu=x_2^*-v_2} = w_1 - w_2 > 0$. The last term is positive since $w_1 > w_2$. But this result contradicts w_a held constant.

Case 2: $x_1^* - v_1 \geq x_2^* - v_2$. From Eq. (7), the difference between the value of w_a when $w = w_1$ and $w = w_2$ is

$$\int_{\mu_L}^{x_2^*-v_2} r(G(x_2^* - \mu) - G(x_1^* - \mu))f(\mu) d\mu + \int_{x_2^*-v_2}^{x_1^*-v_1} (r(1 - G(x_1^* - \mu)) - w_2)f(\mu) d\mu + \int_{x_1^*-v_1}^{\mu_H} (w_1 - w_2)f(\mu) d\mu.$$

The first term is nonnegative since $x_1^* < x_2^*$ from our assumption. The second term is also nonnegative because $G(\cdot)$ is increasing and $(r(1 - G(x_1^* - \mu)) - w_2)|_{\mu=x_2^*-v_2} \geq 0$. The last term is positive since $w_1 > w_2$. But this result contradicts w_a held constant. Hence, x^* must be increasing in w , concluding the proof of Part 2.

To prove Part 3, note that when $w_a = w$, $\frac{d\Pi^r(x)}{dx} = w - w_a = 0$ for $x \in [0, y^*(\mu_L)]$ and $\frac{d\Pi^r(x)}{dx} < 0$ for $x > y^*(\mu_L)$. This implies that the retailer is indifferent in ordering any quantity on the $[0, y^*(\mu_L)]$ interval before the forecast update. The retailer's total purchase quantity under a dual purchase contract is $x^* + (y^*(X) - x^*)^+$, where $y^*(\mu_L)$ is the highest advance purchase quantity when $w_a = w$. Hence, $x^* + (y^*(X) - x^*)^+ = y^*(X)$, which is also the retailer's total purchase quantity under a wholesale price contract. This result implies that the retailer's profit is the same under the two contracts when $w_a = w$.

Part 4 is an immediate consequence of Parts 1 and 2. \square

Proof of Theorem 5. First, we convert the manufacturer's optimization problem in Eq. (8) into an equivalent formulation where the manufacturer sets the quantities. To do so, first note from Theorem 4 Parts 1 and 2 that there is a one-to-one correspondence between the advance purchase price and advance purchase quantity because x^* is decreasing in w_a . Second, note from Theorem 4 Parts 1 and 2 that $x^* \geq y^*(\mu_H)$ when $w_a \in [c_a, \tau(w)]$ and $x^* \in (y^*(\mu_L), y^*(\mu_H))$ when $w_a \in (\tau(w), w)$. Finally, note from Theorem 4 Part 3 that the manufacturer's profit is constant for $x^* \in [0, y^*(\mu_L)]$ when $w_a = w$. Therefore, we consider $x^* = y^*(\mu_L)$ for $w_a = w$. Defining $v \equiv G^{-1}\left(\frac{r-w}{r}\right)$ and substituting for w_a using Eqs. (6) and (7), the manufacturer's expected profit in Eq. (8) can equivalently be written as

$$\Pi_{dp}^m(z) = \begin{cases} \int_{\mu_L}^{x^*-v} (r(1 - G(x^* - \mu))x^* - c(x^* - z)^+)f(\mu) d\mu \\ \quad + \int_{x^*-v}^{\mu_H} (w(\mu + v) - c(\mu + v - z)^+)f(\mu) d\mu - c_a z, & x^* \in [y^*(\mu_L), y^*(\mu_H)), \\ \int_{\mu_L}^{\mu_H} r(1 - G(x^* - \mu))x^*f(\mu) d\mu - c_a z - c(x^* - z)^+, & x^* \geq y^*(\mu_H). \end{cases} \quad (23)$$

To prove Part 1 for $w_a \in [c_a, \tau(w)]$, we show that $\Pi_{dp}^m(z)$ is strictly concave in z for any $x^* \geq y^*(\mu_H)$. When $z < x^*$, the objective function is linearly increasing in z because $\frac{d\Pi_{dp}^m(z)}{dz} = c - c_a > 0$. $\Pi_{dp}^m(z)$ is continuous at $z = x^*$. When $z > x^*$, the objective function is linearly decreasing in z because $\frac{d\Pi_{dp}^m(z)}{dz} = -c_a < 0$. Hence, strict concavity in z holds.

To prove Part 1 for $w_a \in (\tau(w), w]$, we show that $\Pi_{dp}^m(z)$ is strictly concave in z for any $x^* \in [y^*(\mu_L), y^*(\mu_H))$. When $z < x^*$, the objective function is linearly increasing in z because $\frac{d\Pi_{dp}^m(z)}{dz} = c - c_a > 0$. $\Pi_{dp}^m(z)$ is continuous at $z = x^*$. When $z > x^*$, we have $\frac{d\Pi_{dp}^m(z)}{dz} = c(1 - F(z - v)) - c_a$. Hence, $\frac{d\Pi_{dp}^m(x^*, w, z)}{dz} \leq c - c_a$ and $\frac{d^2\Pi_{dp}^m(z)}{dz^2} = -cf(z - v) < 0$ proving the strict concavity in z .

To prove Part 2, we first show that $z_{dp}^* = x^*$ when $w_a \in [c_a, \tau(w)]$. Note from the proof of Part 1 that when $x^* \geq y^*(\mu_H)$, $\Pi_{dp}^m(z)$ is increasing in z for $z < x^*$ and decreasing in z for $z > x^*$. Hence, $z_{dp}^* = x^*$.

Next we show that $z_{dp}^* = \max\{x^*, F^{-1}(\frac{c-c_a}{c}) + G^{-1}(\frac{r-w}{r})\}$ for $w_a \in (\tau(w), w]$. Note from the proof of Part 1 that when $x^* \in [y^*(\mu_L), y^*(\mu_H)]$, $\Pi_{dp}^m(x^*, w, z)$ is increasing in z for $z < x^*$. Hence, $z_{dp}^* \geq x^*$. When $z > x^*$, we have $\frac{d\Pi_{dp}^m(z)}{dz} = c(1 - F(z - v)) - c_a$. When $z < F^{-1}(\frac{c-c_a}{c}) + v$, the objective function is increasing and when $z > F^{-1}(\frac{c-c_a}{c}) + v$, the objective function is decreasing. Hence, $z_{dp}^* = \max\{x^*, F^{-1}(\frac{c-c_a}{c}) + v\}$.

Next we show that $z_{dp}^* \geq F^{-1}(\frac{c-c_a}{c}) + G^{-1}(\frac{r-w}{r})$ for all w_a . Note first that $F^{-1}(\frac{c-c_a}{c}) + v \leq \mu_H + v = y^*(\mu_H)$ since the forecast update X is distributed over $[\mu_L, \mu_H]$. When $w_a \in [c_a, \tau(w)]$, we have from Part 2 that $z_{dp}^* = x^* \geq y^*(\mu_H) \geq F^{-1}(\frac{c-c_a}{c}) + v$. When $w_a \in (\tau(w), w]$, we have $z_{dp}^* \geq F^{-1}(\frac{c-c_a}{c}) + v$ from closed form solution of z_{dp}^* in Part 2. \square

Next, we provide the following lemma as an auxiliary proposition to simplify the proofs of Theorems 6–8.

Lemma 2. Let $w_\ell \equiv r(1 - G(v_\ell))$ where v_ℓ satisfies $r(1 - G(v_\ell)) \left[1 - \frac{g(v_\ell)(\mu_L + v_\ell)}{1 - G(v_\ell)}\right] = 0$.

1. When $w \leq w_\ell$, $\Pi_{dp}^m(w_a, z_{dp}^*)$ is increasing in w_a in the interval $[c_a, \tau(w)]$.
2. When $w \leq w_\ell$, $\Pi_{dp}^m(w_a, z_{dp}^*)$ is increasing in w_a in the interval $(\tau(w), w]$. When $w > w_\ell$, $\Pi_{dp}^m(w_a, z_{dp}^*)$ is decreasing in w_a in the interval $[w_a^1, w]$ where $w_a^1 \in (\tau(w), w)$ and defined in the proof.
3. $w^* > w_\ell$.

Proof of Lemma 2. Before we prove the lemma we convert the manufacturer’s optimization problem in Eq. (8) into an equivalent formulation where the manufacturer sets the quantities. It is shown in the beginning of the proof of Theorem 5 that the conversion leads to Eq. (23). Next we substitute z_{dp}^* found in Theorem 5 into Eq. (23). We have

$$\Pi_{dp}^m(x^*, z_{dp}^*) = \begin{cases} \int_{\mu_L}^{x^*-v} A(x^*, \mu) f(\mu) d\mu + \int_{x^*-v}^{\mu_H} (w(\mu + v) - c_a x^*) f(\mu) d\mu \\ \quad - \int_{F^{-1}(\frac{c-c_a}{c})}^{\mu_H} c(\mu + v - x^*) f(\mu) d\mu, & x^* \in [y^*(\mu_L), F^{-1}(\frac{c-c_a}{c}) + v], \\ \int_{\mu_L}^{x^*-v} A(x^*, \mu) f(\mu) d\mu \\ \quad + \int_{x^*-v}^{\mu_H} ((w - c)(\mu + v) + (c - c_a)x^*) f(\mu) d\mu, & x^* \in (F^{-1}(\frac{c-c_a}{c}) + v, y^*(\mu_H)), \\ \int_{\mu_L}^{\mu_H} A(x^*, \mu) f(\mu) d\mu, & x^* \geq y^*(\mu_H), \end{cases} \tag{24}$$

where $A(x^*, \mu) \equiv (r(1 - G(x^* - \mu)) - c_a)x^*$ and $\frac{dA(x^*, \mu)}{dx^*} = r(1 - G(x^* - \mu)) \left(1 - \frac{g(x^* - \mu)x^*}{1 - G(x^* - \mu)}\right) - c_a$. Note that $\Pi_{dp}^m(x^*, z_{dp}^*)$ is continuous in x^* everywhere.

To prove Part 1, we show that $\Pi_{dp}^m(x^*, z_{dp}^*)$ is decreasing in x^* for $x^* \geq y^*(\mu_H)$. To do so, we show that $\frac{d\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} < 0$ for all $x^* > y^*(\mu_H)$. Note that $x^* > y^*(\mu_H) = \mu_H + G^{-1}(\frac{r-w}{r}) \geq \mu_H + G^{-1}(\frac{r-w_\ell}{r}) = \mu_H + v_\ell$. The second inequality is from $w \leq w_\ell$ and $G(\cdot)$ is increasing. The last equality is from the definition of v_ℓ stated in the Lemma. Hence, $x^* - \mu_H \leq v_\ell$ for all $x^* > y^*(\mu_H)$. This result combined with $G(\cdot)$ having IFR implies that $\frac{dA(x^*, \mu_H)}{dx^*} < r(1 - G(v_\ell)) \left(1 - \frac{g(v_\ell)(\mu_H + v_\ell)}{1 - G(v_\ell)}\right) \leq 0$, where the last inequality is from the definition of v_ℓ and $\mu_H > \mu_L$. Note that $\frac{dA(x^*, \mu)}{dx^*} < 0$ for all $\mu \in [\mu_L, \mu_H]$ because it is nondecreasing in μ . This completes the proof of Part 1.

To prove Part 2, we first show that $\Pi_{dp}^m(w_a, z_{dp}^*)$ is increasing in w_a in the interval $(\tau(w), w]$ when $w \leq w_\ell$. We equivalently show that $\Pi_{dp}^m(x^*, z_{dp}^*)$ is decreasing in x^* for $x^* \in [y^*(\mu_L), y^*(\mu_H)]$. We have

$$\frac{d\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} = \begin{cases} \int_{\mu_L}^{x^*-v} r(1 - G(x^* - \mu)) \left(1 - \frac{g(x^* - \mu)x^*}{1 - G(x^* - \mu)}\right) f(\mu) d\mu, & x^* \in [y^*(\mu_L), F^{-1}(\frac{c-c_a}{c}) + v], \\ \int_{\mu_L}^{x^*-v} r(1 - G(x^* - \mu)) \left(1 - \frac{g(x^* - \mu)x^*}{1 - G(x^* - \mu)}\right) f(\mu) d\mu \\ \quad + c(1 - F(x^* - v)) - c_a, & x^* \in (F^{-1}(\frac{c-c_a}{c}) + v, y^*(\mu_H)). \end{cases} \tag{25}$$

We have $c(1 - F(x^* - v)) - c_a < 0$, when $x^* \in (F^{-1}(\frac{c-c_a}{c}) + v, y^*(\mu_H))$. Hence,

$$\frac{\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} \leq \int_{\mu_L}^{x^*-v} r(1 - G(x^* - \mu)) \left(1 - \frac{g(x^* - \mu)x^*}{1 - G(x^* - \mu)}\right) f(\mu) d\mu$$

for $x^* \in [y^*(\mu_L), y^*(\mu_H)]$. Notice that $x^* - \mu > v \geq v_\ell$ for all $\mu \in [\mu_L, x^* - v]$, where the second inequality follows from $w \leq w_\ell$. This result combined with $G(\cdot)$ having IFR implies

$$r(1 - G(x^* - \mu)) \left(1 - \frac{x^*g(x^* - \mu)}{1 - G(x^* - \mu)} \right) < r(1 - G(v_\ell)) \left[1 - \frac{(\mu_L + v_\ell)g(v_\ell)}{1 - G(v_\ell)} \right] = 0$$

for all $\mu \in [\mu_L, x^* - v]$. Hence $\frac{d\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} < 0$ when $x^* \in (y^*(\mu_L), y^*(\mu_H))$.

Next we show that when $w > w_\ell$, $\Pi_{dp}^m(w_a, z_{dp}^*)$ is decreasing w_a in the interval $[w_a^1, w]$ for some $w_a^1 \in (\tau(w), w)$. We equivalently prove that $\Pi_{dp}^m(x^*, z_{dp}^*)$ is increasing in x^* on $[y^*(\mu_L), x_1^*]$ for some $x_1^* \in (y^*(\mu_L), y^*(\mu_H))$. Let $x_1^* = \min\{\mu_L + v_\ell, F^{-1}(\frac{c-c_a}{c}) + v\}$. Note that $x_1^* \in (y^*(\mu_L), y^*(\mu_H))$ since $\mu_L + v_\ell > y^*(\mu_L)$ for $w > w_\ell$ and $F^{-1}(\frac{c-c_a}{c}) \in (\mu_L, \mu_H)$ for $c_a \in (0, c)$. Since $x_1^* \leq F^{-1}(\frac{c-c_a}{c}) + v$, from Eq. (25),

$$\frac{d\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} = \int_{\mu_L}^{x^*-v} r(1 - G(x^* - \mu)) \left(1 - \frac{x^*g(x^* - \mu)}{1 - G(x^* - \mu)} \right) f(\mu) d\mu \quad \text{for all } x^* \in (y^*(\mu_L), x_1^*]. \tag{26}$$

Note that $x^* \leq x_1^* \leq \mu_L + v_\ell$ for all $x^* \in (y^*(\mu_L), x_1^*]$, where the second inequality is from the definition of x_1^* . This implies that $x^* - \mu_L \leq v_\ell$ for all $x^* \in (y^*(\mu_L), x_1^*]$. Hence, for all $x^* \in (y^*(\mu_L), x_1^*]$, we have $x^* - \mu < v_\ell$ for all $\mu \in (\mu_L, x^* - v]$. This result combined with $G(\cdot)$ having IFR implies that for all $x^* \in (y^*(\mu_L), x_1^*]$,

$$r(1 - G(x^* - \mu)) \left(1 - \frac{x^*g(x^* - \mu)}{1 - G(x^* - \mu)} \right) > r(1 - G(v_\ell)) \left[1 - \frac{g(v_\ell)(\mu_L + v_\ell)}{1 - G(v_\ell)} \right] = 0$$

for all $\mu \in (\mu_L, x^* - v]$. Hence, from Eq. (26), $\frac{d\Pi_{dp}^m(x^*, z_{dp}^*)}{dx^*} > 0$ for all $x^* \in (y^*(\mu_L), x_1^*]$.

To prove Part 3, recall that $\bar{\mu} > \mu_L$ and compare the definition of w_ℓ in Theorem 6 with the definition of w^* in Theorem 2. This leads to $w_\ell < w^*$. \square

Proof of Theorem 6. To prove Part 1, we solve $\max_{w_a \in [c_a, w]} \Pi_{dp}^m(w_a, z_{dp}^*)$. Since $w \leq w_\ell$, Lemma 2 Part 1 implies that $\Pi_{dp}^m(w_a, z_{dp}^*)$ is increasing in the interval $[c_a, \tau(w)]$ and maximized at $\tau(w)$. $\Pi_{dp}^m(w_a, z_{dp}^*)$ is continuous everywhere. From Lemma 2 Part 2, $\Pi_{dp}^m(w_a, z_{dp}^*)$ is increasing on $(\tau(w), w]$ and maximized at $w_a = w$. Hence, $w_a^* = w$ when $w \leq w_\ell$.

To prove Part 2, note from Lemma 2 Part 2 that when $w > w_\ell$, there exists a $w_a^1 \in (\tau(w), w)$ such that $\Pi_{dp}^m(w_a, z_{dp}^*)$ is decreasing in $[w_a^1, w]$. Therefore, $w_a^* < w$. This concludes our proof. \square

Proof of Theorem 7. When $w \leq w_\ell$, from Theorem 6 Part 1, we have $w_a^* = w$. Because the two contracts are equivalent in this case, the manufacturer’s profits under the two contracts are equal. When $w > w_\ell$, note that $\Pi_{dp}^m(w_a^*, z_{dp}^*) \geq \Pi_{dp}^m(w_a^1, z_{dp}^*) > \Pi_{dp}^m(w, z_{dp}^*) = \Pi^m(w, z^*)$, where w_a^1 is as defined in Lemma 2 Part 2. The first inequality is from the definition of w_a^* . The second inequality is from Lemma 2 Part 2. The equality is from above. Hence, $\Pi_{dp}^m(w_a^*, w, z_{dp}^*) > \Pi^m(w, z^*)$. \square

Proof of Theorem 8. Note from Lemma 2 Part 3 that $w^* > w_\ell$. This result combined with the result of Theorem 7 implies that $\Pi^m(w^*, z^*) < \Pi_{dp}^m(w_a^*, w^*, z_{dp}^*) \leq \Pi_{dp}^*$. The second inequality holds because (w_a^*, w^*) is a feasible solution to (10). This completes the proof. \square

A.3. Proofs of theorems in Section 6

Proof of Theorem 9. The proof is analogous to the proof of Theorem 3. To verify, it suffices to replace x^* with z^{cs} , $y^*(\mu)$ with $y^{cs}(\mu)$, w with c and w_a with c_a in the proof of Theorem 3. \square

Proof of Theorem 10. The proofs of Parts 1, 2, 3 and 4 of this theorem are analogous to the proofs of same numbered Parts of Theorem 4. To verify, it suffices to replace x^* with z^{cs} , $y^*(\mu)$ with $y^{cs}(\mu)$, w with c and w_a with c_a in the proof of Theorem 4.

To prove Part 5, we consider two cases.

Case 1: $c_a \in [0, \tau^{cs}(c)]$. We have, $z^{cs} \geq y^{cs}(\mu_H) = \mu_H + G^{-1}\left(\frac{r-c}{r}\right) > \mu_H + G^{-1}\left(\frac{r-w}{r}\right) > F^{-1}\left(\frac{c-c_a}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)$. The first inequality is from Part 1. The equality is from the definition of $y^{cs}(\mu_H)$ in Theorem 9 Part 1. The second inequality is from $w < r$. The third inequality holds because X is distributed on $[\mu_L, \mu_H]$ and $c_a < c$.

Case 2: $c_a \in (\tau^{cs}(c), c)$. From Eq. (14) in Part 2, we have,

$$c\left(1 - F\left(z^{cs} - G^{-1}\left(\frac{r-c}{r}\right)\right)\right) - c_a = -r \left[\int_{\mu_L}^{z^{cs} - G^{-1}\left(\frac{r-c}{r}\right)} (1 - G(z^{cs} - \mu))f(\mu) d\mu \right]. \tag{27}$$

Note that the right-hand side of (27) is strictly negative since $z^{cs} > y^{cs}(\mu_L) = \mu_L + G^{-1}\left(\frac{r-c}{r}\right)$ when $c_a \in (\tau(c), c)$ (Part 2). Note also that $z = F^{-1}\left(\frac{c_a-c}{c}\right) + G^{-1}\left(\frac{r-c}{r}\right)$ satisfies

$$c\left(1 - F\left(z - G^{-1}\left(\frac{r-c}{r}\right)\right)\right) - c_a = 0. \tag{28}$$

Hence, the right-hand side of (27) is strictly smaller than the right-hand side of (28). This is enough to conclude that $z^{cs} > F^{-1}\left(\frac{c_a-c}{c}\right) + G^{-1}\left(\frac{r-c}{r}\right)$ since $F(\cdot)$ is increasing. Therefore, we have $z^{cs} > F^{-1}\left(\frac{c_a-c}{c}\right) + G^{-1}\left(\frac{r-c}{r}\right) > F^{-1}\left(\frac{c_a-c}{c}\right) + G^{-1}\left(\frac{r-w}{r}\right)$, where the second inequality is from $c < w$ and $G(\cdot)$ is increasing. \square

Proof of Theorem 11. We first find expressions for total supply chain profit under each contract. Recall from Eq. (2) that given a wholesale price contract, the retailer orders $y^*(\mu)$ units after the forecast update. Hence, her optimal expected profit before the forecast update, as defined in Eq. (1), is $E\Pi^r(y^*(X), X)$. The manufacturer’s optimal advance production quantity, as characterized in Theorem 1 Part 1, is z^* . Therefore, the manufacturer’s expected profit, as defined in Eq. (3), is $\Pi^m(w, z^*)$. Hence, the supply chain profit under a wholesale price contract is equal to

$$E\Pi^r(y^*(X), X) + \Pi^m(w, z^*) = \int_{\mu_L}^{\mu_H} [rE(\min(y^*(\mu), \mu + \epsilon)) - c_a z^* - c(y^*(\mu) - z^*)^+] f(\mu) d\mu. \tag{29}$$

Next we find the total supply chain profit under the dual purchase contract. We consider only the case where $w_a \in (\tau(w), w)$. We define $v \equiv G^{-1}\left(\frac{r-w}{r}\right)$ and write the retailer’s expected profit before the forecast update: $\int_{\mu_L}^{x^*-v} (rE(\min(x^*, \mu + \epsilon)) - w_a x^*) f(\mu) d\mu + \int_{x^*-v}^{\mu_H} [rE(\min(y^*(\mu), \mu + \epsilon)) - w_a x^* - w(y^*(\mu) - x^*)] f(\mu) d\mu$. From Eq. (8), the manufacturer’s expected profit is $w_a x^* + wE(y^*(X) - x^*)^+ - c_a z_{dp}^* - cE(\max\{x^*, y^*(X)\} - z_{dp}^*)^+$. Hence, the supply chain profit under the dual purchase contract is

$$\int_{\mu_L}^{x^*-v} (rE(\min(x^*, \mu + \epsilon)) - c_a z_{dp}^* - c(x^* - z_{dp}^*)^+) f(\mu) d\mu + \int_{x^*-v}^{\mu_H} (rE(\min(y^*(\mu), \mu + \epsilon)) - c_a z_{dp}^* - c(y^*(\mu) - z_{dp}^*)^+) f(\mu) d\mu. \tag{30}$$

Recall from Theorem 4 Part 2 that when $w_a \in (\tau(w), w)$, we have $x^* \in (y^*(\mu_L), y^*(\mu_H))$. We show that there exists an $x_o^* \in (y^*(\mu_L), y^*(\mu_H))$ such that the supply chain profit under the dual purchase contract that implements x_o^* is higher than the supply chain profit under the wholesale price contract. For any $x^* \in (y^*(\mu_L), y^*(\mu_H))$, the difference between the supply chain profit under the dual purchase contract and the wholesale price contract is found by subtracting (29) from (30) and is equal to

$$\int_{x^*-v}^{\mu_H} \left\{ \underbrace{c_a(z^* - z_{dp}^*) + c((y^*(\mu) - z^*)^+ - (y^*(\mu) - z_{dp}^*)^+)}_{M(\mu, x^*)} \right\} f(\mu) d\mu + \int_{\mu_L}^{x^*-v} \left\{ \underbrace{r \left(\int_{y^*(\mu)-\mu}^{x^*-\mu} (\mu + \epsilon - y^*(\mu)) g(\epsilon) d\epsilon \right)}_{N(\mu, x^*)} \right\} f(\mu) d\mu + \underbrace{(x^* - y^*(\mu))r(1 - G(x^* - \mu)) - c_a(z_{dp}^* - z^*) - c((x^* - z_{dp}^*)^+ - (y^*(\mu) - z^*)^+)}_{O(\mu, x^*)} f(\mu) d\mu. \tag{31}$$

We first show that $M(\mu, x^*) \geq 0$ for all $\mu \in [x^* - v, \mu_H]$ for any $x^* \in (y^*(\mu_L), y^*(\mu_H))$. Recall from **Theorem 5** Part 2 that $z_{dp}^* = \max\{x^*, F^{-1}(\frac{c-c_a}{c}) + G^{-1}(\frac{r-w}{r})\}$. Noting from **Theorem 1** Part 1 that $z^* = F^{-1}(\frac{c-c_a}{c}) + G^{-1}(\frac{r-w}{r})$, we have $z_{dp}^* = \max\{x^*, z^*\}$. If $z_{dp}^* = z^*$, then we have $M(\mu, x^*) = 0$. Otherwise, we recall that $y^*(\mu) \geq x^*$ for all $\mu \in [x^* - v, \mu_H]$ to conclude that $y^*(\mu) \geq x^* = z_{dp}^* > z^*$. This gives $M(\mu, x^*) = (c - c_a)(z_{dp}^* - z^*) > 0$. Hence, we have $\int_{x^*-v}^{\mu_H} M(\mu, x^*) f(\mu) d\mu \geq 0$. Next we prove that the rest of Eq. (31) is positive for some $x_o^* \in (y^*(\mu_L), y^*(\mu_H))$. If $\mu_L + G^{-1}(\frac{r-c}{r}) < y^*(\mu_H)$, then let $x_o^* \equiv \mu_L + G^{-1}(\frac{r-c}{r})$; otherwise, let $x_o^* \equiv y^*(\mu_H) - \delta$ for some small $\delta > 0$. Observe that $N(\mu, x_o^*) > 0$ for all $\mu \in [\mu_L, x_o^* - v]$ because $y^*(\mu) = \mu + v < x_o^*$ in this interval. To prove that $O(\mu, x_o^*) > 0$ for all $\mu \in (\mu_L, x_o^* - v]$, we first show that the first multiplier in $O(\mu, x_o^*)$ is positive. That is, $x_o^* - y^*(\mu) > 0$ for all $\mu \in [\mu_L, x_o^* - v]$. This follows from the definition of $y^*(\mu)$ in Eq. (2) because $y^*(\mu) = \mu + v < x_o^*$ for all $\mu \in [\mu_L, x_o^* - v]$. Next we show that the second multiplier of $O(\mu, x_o^*)$ is positive for all $\mu \in (\mu_L, x_o^* - v]$. To do so, we first show that

$$c(x_o^* - y^*(\mu)) \geq c_a(z_{dp}^* - z^*) + c\left((x_o^* - z_{dp}^*)^+ - (y^*(\mu) - z^*)^+\right) \tag{32}$$

for all $\mu \in (\mu_L, x_o^* - v]$. To do so, recall from the proof of $M(\mu, x^*) \geq 0$ that $z_{dp}^* = \max\{x^*, z^*\}$. First consider $z_{dp}^* = z^*$ and recall that $x_o^* \geq y^*(\mu)$ for all $(\mu_L, x_o^* - v]$. Using this and $z_{dp}^* = \max\{x_o^*, z^*\}$, we have $z^* = z_{dp}^* \geq x_o^* \geq y^*(\mu)$. This gives that the right-hand side of (32) is zero. The left-hand side is non-negative since $x_o^* \geq y^*(\mu)$ for all $(\mu_L, x_o^* - v]$. Hence, (32) holds when $z_{dp}^* = z^*$. When $z_{dp}^* = x_o^* > z^*$, the right-hand side can be written as $c_a(x_o^* - z^*) - c(y^*(\mu) - z^*)^+$. From this and $c > c_a$, we conclude that (32) holds. Next we use (32) to conclude that

$$O(\mu, x_o^*) > (x_o^* - y^*(\mu))r(1 - G(x_o^* - \mu)) - c(x_o^* - y^*(\mu)) = (x_o^* - y^*(\mu))r(1 - G(x_o^* - \mu) - c).$$

Hence, it is enough to show that $(r(1 - G(x_o^* - \mu)) - c) > 0$ for all $\mu \in (\mu_L, x_o^* - v]$. This is equivalent to proving that $G(x_o^* - \mu) < \frac{r-c}{r}$ for all $\mu \in (\mu_L, x_o^* - v]$, which is equivalent to $x_o^* < \mu + G^{-1}(\frac{r-c}{r})$ for all $\mu \in (\mu_L, x_o^* - v]$. The latter holds by choice of x_o^* . \square

A.4. Proofs of theorems in Section 7 and required technical lemmas

Proof of Theorem 12. The proof that w^* satisfies the first order condition in the theorem is analogous to the proof of **Theorem 2** Part 1. Note that when $F(\cdot)$ is a degenerate distribution such that $\text{Prob}\{X = \mu_L\} = 1$, then $\bar{\mu} = \mu_L$, the lowest possible forecast update. In this case, the optimal wholesale price is w_L^* .

The proof of Part 2 is analogous to the proof of **Theorem 2** Part 2. Note that $w^* \geq w_L^*$ since $w^* = w_L^*$ when $\bar{\mu} = \mu_L$. The proof of Part 3 is analogous to the proof of **Theorem 2** Part 3. This concludes the proof. \square

Proof of Theorem 13. First we provide the following lemma as an auxiliary proposition to simplify the proof of **Theorem 13**.

Lemma 3

When $c_a > c$,

1. $\Pi_{dp}^m(w_a)$ in Eq. (16) is increasing in w_a in the interval $[c, \tau(w)]$ for any $w \leq w_L^*$.
2. When $w \leq w_L^*$, $\Pi_{dp}^m(w_a)$ is increasing in w_a in the interval $(\tau(w), w]$. When $w > w_L^*$, $\Pi_{dp}^m(w_a)$ is decreasing in w_a in the interval $[w_a^2, w]$ for some $w_a^2 \in (\tau(w), w)$.
3. $w_L^* > w_L$.

Proof of Lemma 3. First, we convert the manufacturer’s optimization problem in Eq. (16) into an equivalent formulation where the manufacturer sets the quantities. To do so, first note from **Theorem 4** Parts 1 and 2 that there is a one-to-one correspondence between the advance purchase price and advance purchase quantity when $w_a \in [c, w)$ because x^* is decreasing in w_a . Second, note from **Theorem 4** Parts 1 and 2 that $x^* \geq y^*(\mu_H)$ when $w_a \in [c, \tau(w)]$ and $x^* \in (y^*(\mu_L), y^*(\mu_H))$ when $w_a \in (\tau(w), w)$. Finally, note from **Theorem 4** Part 2 that manufacturer’s profit is constant for $x^* \in [0, y^*(\mu_L)]$ when $w_a = w$. Therefore, we consider $x^* = y^*(\mu_L)$ for

$w_a = w$. Defining $v \equiv G^{-1}\left(\frac{r-w}{r}\right)$ and substituting for w_a using Eqs. (6) and (7), the manufacturer’s expected profit in Eq. (16) can equivalently be written as

$$\Pi_{dp}^m(x^*) = \begin{cases} (w - c) \int_{x^*-v}^{\mu_H} (\mu + v) f(\mu) d\mu \\ \quad + \int_{\mu_L}^{x^*-v} A(x^*, \mu) f(\mu) d\mu, & x^* \in [y^*(\mu_L), y^*(\mu_H)], \\ \int_{\mu_L}^{\mu_H} A(x^*, \mu) f(\mu) d\mu, & x^* \geq y^*(\mu_H), \end{cases} \quad (33)$$

where $A(x^*, \mu) \equiv (r(1 - G(x^* - \mu)) - c)x^*$ and $\frac{dA(x^*, \mu)}{dx^*} = r(1 - G(x^* - \mu))\left(1 - \frac{g(x^* - \mu)x^*}{1 - G(x^* - \mu)}\right) - c$.

To prove Part 1, we show that $\Pi_{dp}^m(x^*)$ is decreasing in x^* for $x^* \geq y^*(\mu_H)$. To do so, we show $\frac{d\Pi_{dp}^m(x^*)}{dx^*} < 0$ for all $x^* > y^*(\mu_H)$. Note that $x^* > y^*(\mu_H) = \mu_H + G^{-1}\left(\frac{r-w}{r}\right) \geq \mu_H + G^{-1}\left(\frac{r-w_L^*}{r}\right) = \mu_H + v_L^*$. The second inequality is from $w \leq w_L^*$ and $G(\cdot)$ is increasing. The last equality is from the definition of v_L^* in Theorem 12 Part 1. Hence, $x^* - \mu_H > v_L^*$ for all $x^* > y^*(\mu_H)$. This result combined with $G(\cdot)$ having IFR implies that $\frac{dA(x^*, \mu_H)}{dx^*} < r(1 - G(v_L^*))\left(1 - \frac{g(v_L^*)(\mu_H + v_L^*)}{1 - G(v_L^*)}\right) - c < 0$, where the last inequality is from the definition of v_L^* in Theorem 12 Part 1 and $\mu_L < \mu_H$. Note that $\frac{dA(x^*, \mu)}{dx^*} < 0$ for all $\mu \in [\mu_L, \mu_H]$ because it is nondecreasing in μ . This completes the proof of Part 1.

To prove Part 2, we equivalently show that $\Pi_{dp}^m(x^*)$ is decreasing in x^* for $x^* \in [y^*(\mu_L), y^*(\mu_H)]$ when $w \leq w_L^*$. We have,

$$\frac{d\Pi_{dp}^m(x^*)}{dx^*} = \int_{\mu_L}^{x^*-v} \frac{dA(x^*, \mu)}{dx^*} f(\mu) d\mu. \quad (34)$$

Hence, $\Pi_{dp}^m(x^*)$ is decreasing if $\frac{dA(x^*, \mu)}{dx^*} < 0$ for all $\mu \in [\mu_L, x^* - v]$. To prove this, note that $x^* - \mu > G^{-1}\left(\frac{r-w}{r}\right) \geq G^{-1}\left(\frac{r-w_L^*}{r}\right) = v_L^*$ for all $\mu \in [\mu_L, x^* - v]$. The first inequality is from the set μ belongs to. The second inequality is from $w \leq w_L^*$. The equality is from the definition of v_L^* in Theorem 2 Part 1. This result combined with $G(\cdot)$ having IFR implies that $\frac{dA(x^*, \mu)}{dx^*} < r(1 - G(v_L^*))\left(1 - \frac{g(v_L^*)(\mu_L + v_L^*)}{1 - G(v_L^*)}\right) - c = 0$ for all $\mu \in [\mu_L, x^* - v]$, where the last equality is by the definition of v_L^* . Hence, $\frac{d\Pi_{dp}^m(x^*)}{dx^*} < 0$ for all $x^* \in (y^*(\mu_L), y^*(\mu_H))$.

Next we prove that when $w > w_L^*$, $\Pi_{dp}^m(w_a)$ is decreasing in w_a on $[w_a^2, w]$ for some $w_a^2 \in (\tau(w), w)$. To do so, we equivalently prove that $\Pi_{dp}^m(x^*)$ is increasing in x^* on $[y^*(\mu_L), x_2^*]$ for some $x_2^* \in (y^*(\mu_L), y^*(\mu_H))$. Let $x_2^* \equiv \mu_L + G^{-1}\left(\frac{r-w_L^*}{r}\right)$ if $\mu_L + G^{-1}\left(\frac{r-w_L^*}{r}\right) < y^*(\mu_H)$; otherwise let $x_2^* \equiv y^*(\mu_H) - \delta$ for some small $\delta > 0$. We first show that $\left.\frac{d\Pi_{dp}^m(x^*)}{dx^*}\right|_{x^*=x_2^*} > 0$. From Eq. (34), it is sufficient to show that $\left.\frac{dA(x^*, \mu)}{dx^*}\right|_{x^*=x_2^*} > 0$ for all $\mu \in (\mu_L, x_2^* - v]$. Note that $x_2^* - \mu < G^{-1}\left(\frac{r-w_L^*}{r}\right)$ for all $\mu \in (\mu_L, x_2^* - v]$. This result combined with $G(\cdot)$ having IFR implies that

$$\left.\frac{dA(x^*, \mu)}{dx^*}\right|_{x^*=x_2^*} > r(1 - G(v_L^*))\left(1 - \frac{g(v_L^*)(\mu_L + v_L^*)}{1 - G(v_L^*)}\right) - c = 0$$

for all $\mu \in (\mu_L, x_2^* - v]$. Note that $\frac{dA(x^*, \mu)}{dx^*}$ is decreasing in x^* . Therefore, Eq. (34) implies that $\frac{d\Pi_{dp}^m(x^*)}{dx^*} > 0$ for all $x^* \in (y^*(\mu_L), x_2^*]$.

To prove Part 3, we compare the definition of w_ℓ in Theorem 6 with the definition of w_L^* in Theorem 12, which leads to $w_\ell < w_L^*$. \square

Next we prove the theorem. Note that when $w_a = w$, the dual purchase contract and the wholesale price contract are equivalent. We first show that when $w \leq w_L^*$, we have $w_a^* = w$. Since $w \leq w_L^*$, Lemma 3 Part 1 (in Appendix A) implies that $\Pi_{dp}^m(w_a)$ is increasing in the interval $[c, \tau(w)]$ and maximized at $\tau(w)$. $\Pi_{dp}^m(w_a)$ is continuous everywhere. From Lemma 3 Part 2, $\Pi_{dp}^m(w_a)$ is increasing on $(\tau(w), w]$ and maximized at $w_a = w$. We have, $w_a^* = w$. Therefore, the dual purchase contract and wholesale price contract are equivalent when $w \leq w_L^*$. When $w > w_L^*$, note that $\Pi_{dp}^m(w_a^*) \geq \Pi_{dp}^m(w_a^2) > \Pi_{dp}^m(w) = \Pi^m(w)$, where w_a^2 is as defined in Lemma 3 Part 2. The first inequality is from the definition of w_a^* . The second inequality is from Lemma 3 Part 2. The equality is from the equivalence of these contracts when $w_a = w$. \square

A.5. Proofs of theorems in Section 8 and required technical lemmas

Proof of Theorem 14. The manufacturer’s profit is defined in Eq. (16). The variance of this profit is $\text{Var}_{dp}(w_a) = (w - c)^2 \text{Var}[(y^*(X) - x^*)^+]$.

When $w_a \in [c, \tau(w)]$, variance under the dual purchase contract is zero because $x^* \geq y^*(\mu_L)$ by Theorem 4 Part 1. When $w_a \in (\tau(w), w)$, Theorem 4 Part 2 states that $x^* \in (y^*(\mu_L), y^*(\mu_H))$. By recalling the definition of $y^*(\mu)$ from Eq. (2), we can write x^* as the sum of two constants; i.e., $x^* = \mu_o + G^{-1}(\frac{r-w}{r})$ where $\mu_o \in (\mu_L, \mu_H)$. Then, variance under the dual purchase contract can be written as $(w - c)^2 \text{Var}[(y^*(X) - x^*)^+] = (w - c)^2 \text{Var}(X - \mu_o)^+$. The variance under a wholesale price contract is $\text{Var}((w - c)y^*(X)) = (w - c)^2 \text{Var}(X) = (w - c)^2 \text{Var}(X - \mu_o) \geq (w - c)^2 \text{Var}(X - \mu_o)^+$. When $w_a = w$, the manufacturer’s profit under the dual purchase contract is $(w - c)x^* + (w - c)(y^*(X) - x^*)^+ = (w - c)y^*(X)$. The equality is from Theorem 4 Part 3.

To prove the monotonicity result, note that when $w_a \in (\tau(w), w]$, we have

$$\frac{d\text{Var}_{dp}(w_a)}{dx^*} = -2F(x^* - v) \int_{x^*-v}^{\mu_H} (\mu + v - x^*)f(\mu) d\mu < 0 \text{ for all } x^* \in (y^*(\mu_L), y^*(\mu_H)),$$

where $v \equiv G^{-1}(\frac{r-w}{r})$. Since x^* is decreasing in w_a by Theorem 4 Part 2, $\text{Var}_{dp}(w_a)$ is increasing in w_a . □

Proof of Theorem 15. First we provide the following lemma as an auxiliary proposition to simplify the proof of Theorem 15.

Lemma 4

When $c_a > c$,

1. $\Pi_{ra}^m(w_a)$ is increasing in w_a in the interval $[c, \tau(w)]$ for any $w \leq w_L^*$.
2. When $k \leq \underline{k}$, $\Pi_{ra}^m(w_a)$ is increasing in w_a in the interval $(\tau(w), w]$. When $k > \underline{k}$, $\Pi_{ra}^m(w_a)$ is decreasing in w_a in the interval $[w_a^3, w]$ for some $w_a^3 \in (\tau(w), w)$.
3. Under a dual purchase contract with $w_a = w$, the manufacturer’s objective function value is the same as his objective function value under a wholesale price contract regardless of his risk attitude.

Proof of Lemma 4. To prove Part 1, note from Eqs. (16) and (18) that $\Pi_{ra}^m(w_a) = \Pi_{dp}^m(w_a)$ when $w_a \in [c, \tau(w)]$. Hence, from Lemma 3 Part 1 (in Appendix A), $\Pi_{ra}^m(w_a)$ is increasing in w_a on $[c, \tau(w)]$.

To prove Part 2, we convert $\Pi_{ra}^m(w_a)$ in Eq. (18) (when $w_a \in (\tau(w), w)$) into an equivalent formulation where the manufacturer sets the quantity. To do so, first note from Theorem 4 Parts 1 and 2 that there is a one-to-one correspondence between the advance purchase price and advance purchase quantity since x^* is decreasing in w_a . Also note from Theorem 4 Part 2 that $x^* \in (y^*(\mu_L), y^*(\mu_H))$ when $w_a \in (\tau(w), w)$. Finally, note from Theorem 4 Part 3 that manufacturer’s profit is constant for $x^* \in [0, y^*(\mu_L)]$ when $w_a = w$. Therefore, we consider $x^* = y^*(\mu_L)$ for $w_a = w$. Defining $v \equiv G^{-1}(\frac{r-w}{r})$ and substituting for w_a using Eq. (7), $\Pi_{ra}^m(x^*)$ can be written as

$$(w - c) \int_{x^*-v}^{\mu_H} (\mu + v)f(\mu) d\mu + \int_{\mu_L}^{x^*-v} [r(1 - G(x^* - \mu)) - c]x^*f(\mu) d\mu - k(w - c)^2 \text{Var}[(X + v - x^*)^+],$$

where $x^* \in [y^*(\mu_L), y^*(\mu_H)]$. We have,

$$\begin{aligned} & \frac{d\Pi_{ra}^m(x^*)}{dx^*} \\ &= \int_{\mu_L}^{x^*-v} \underbrace{\left(r(1 - G(x^* - \mu)) \left(1 - \frac{x^*g(x^* - \mu)}{1 - G(x^* - \mu)} \right) - c + 2k(w - c)^2 \int_{x^*-v}^{\mu_H} (s - (x^* - v))f(s) ds \right)}_{B(x^*, \mu, k)} f(\mu) d\mu. \end{aligned} \tag{35}$$

Recall from Eq. (2) that $y^*(\mu_L) = \mu_L + v$. Hence,

$$B(y^*(\mu_L), \mu_L, k) = w - c - ry^*(\mu_L)g(v) + 2k(w - c)^2(\bar{\mu} - \mu_L). \tag{36}$$

Similarly, for $x^* > y^*(\mu_L)$, we have,

$$B(x^*, x^* - v, k) = w - c - rx^*g(v) + 2k(w - c)^2 \int_{x^*-v}^{\mu_H} (s - (x^* - v))f(s) ds. \tag{37}$$

The definition of \underline{k} in Theorem 15 and Eq. (36) imply that $B(y^*(\mu_L), \mu_L, \underline{k}) = 0$. $B(y^*(\mu_L), \mu_L, k) \leq 0$ for $k \leq \underline{k}$ since $B(y^*(\mu_L), \mu_L, k)$ is increasing in k from Eq. (36). Note from Eqs. (36) and (37) that $B(x^*, x^* - v, k) < B(y^*(\mu_L), \mu_L, k)$ for all $x^* \in (y^*(\mu_L), y^*(\mu_H))$. Hence, $B(x^*, x^* - v, k) < 0$ for all $x^* \in (y^*(\mu_L), y^*(\mu_H))$. Note that $B(x^*, \mu, k)$ is nondecreasing in μ since $G(\cdot)$ has IFR. Hence, $B(x^*, \mu, k) < 0$ for all $\mu \in [\mu_L, x^* - v]$ and for all $x^* \in (y^*(\mu_L), y^*(\mu_H))$. Therefore, $\frac{d\Pi_{ra}^m(x^*)}{dx^*} < 0$ for all $x^* \in (y^*(\mu_L), y^*(\mu_H))$, which implies that $\Pi_{ra}^m(x^*)$ is decreasing on $[y^*(\mu_L), y^*(\mu_H)]$.

Next we prove that when $k > \underline{k}$, $\Pi_{ra}^m(w_a)$ is decreasing in w_a on $[w_a^3, w]$ for some $w_a^3 \in (\tau(w), w)$. To do so, we equivalently prove that $\Pi_{ra}^m(x^*)$ is increasing on $[y^*(\mu_L), x_3^*]$ for some $x_3^* \in (y^*(\mu_L), y^*(\mu_H))$. The definition of \underline{k} implies that $B(y^*(\mu_L), \mu_L, \underline{k}) = 0$. $B(y^*(\mu_L), \mu_L, k) > 0$ for $k > \underline{k}$ since $B(y^*(\mu_L), \mu_L, k)$ is increasing in k from Eq. (36). Since $B(x^*, \mu_L, k)$ is continuous in x^* , there exists an $x_3^* \in (y^*(\mu_L), y^*(\mu_H))$ such that $B(x_3^*, \mu_L, k) > 0$. Note that $B(x_3^*, \mu, k)$ is nondecreasing in μ since $G(\cdot)$ has IFR. Hence, $B(x_3^*, \mu, k) > 0$ for all μ . Therefore, $\frac{d\Pi_{ra}^m(x^*)}{dx^*} \Big|_{x^*=x_3^*} > 0$. Note from Eq. (35) that $B(x^*, \mu, k)$ is decreasing in x^* . Therefore, $\frac{d\Pi_{ra}^m(x^*)}{dx^*} > 0$ for all $x^* \in (y^*(\mu_L), x_3^*]$, which implies that $\Pi_{ra}^m(x^*)$ is increasing in x^* on $[y^*(\mu_L), x_3^*]$.

To prove Part 3, note from Theorem 4 Part 3 that when $w_a = w$, retailer’s total purchase quantity under the dual purchase contract is equal to her total purchase quantity under the wholesale price contract for any realization of X . Hence, the manufacturer’s profit and its variance are the same too. \square

Next we prove the theorem. To do so, note that the manufacturer’s profit under the wholesale price contract when $c_a > c$ and the forecast update is μ_L : $(w - c)y^*(\mu_L)$. From Theorem 12 and $c_a > c$, we know that this profit function is unimodal in w , with the optimal price w_a^* . Also note that the derivative of $(w - c)y^*(\mu_L)$ with respect to w is $\frac{w-c}{-rg(y^*(\mu_L)-\mu_L)} + y^*(\mu_L) = -\frac{(w-c)-ry^*(\mu_L)g(y^*(\mu_L)-\mu_L)}{rg(y^*(\mu_L)-\mu_L)}$. To prove Part 1, note that when $k > \underline{k}$, $\Pi_{dp}^m(w_a^*) \geq \Pi_{dp}^m(w_a^3) > \Pi_{dp}^m(w) = \Pi^m(w)$, where w_a^3 is as defined in Lemma 4 Part 2 (in Appendix A). The first inequality is from the definition of w_a^* . The second inequality is from Lemma 3 Part 2. The equality is from Lemma 3 Part 3.

To prove the monotonicity result, let $k_2 > k_1$ and w_{a1}^*, w_{a2}^* be the corresponding optimal advance purchase prices. Recall from Lemma 3 Part 1 that $w_{a1}^*, w_{a2}^* \in [\tau(w), w]$. Assume for a contradiction argument $w_{a1}^* < w_{a2}^*$. From Eq. (17), we have $\Pi_{ra}^m(w_{a2}^*) = \Pi_{dp}^m(w_{a2}^*) - k_2 \text{Var}_{dp}(w_{a2}^*) \geq \Pi_{dp}^m(w_{a1}^*) - k_2 \text{Var}_{dp}(w_{a1}^*)$ by the optimality of w_{a2}^* for k_2 . Hence,

$$\Pi_{dp}^m(w_{a2}^*) - \Pi_{dp}^m(w_{a1}^*) \geq k_2[\text{Var}_{dp}(w_{a2}^*) - \text{Var}_{dp}(w_{a1}^*)] > k_1[\text{Var}_{dp}(w_{a2}^*) - \text{Var}_{dp}(w_{a1}^*)].$$

The last inequality is from Theorem 14 and $k_2 > k_1$. This implies $\Pi_{dp}^m(w_{a2}^*) - k_1 \text{Var}_{dp}(w_{a2}^*) > \Pi_{dp}^m(w_{a1}^*) - k_1 \text{Var}_{dp}(w_{a1}^*)$, which contradicts the optimality of w_{a1}^* for k_1 .

To prove Part 2, note Lemma 3 Part 1 states that $\Pi_{ra}^m(w_a)$ is increasing on $[c, \tau(w)]$ and maximized at $\tau(w)$. $\Pi_{ra}^m(w_a)$ is continuous everywhere. Lemma 3 Part 2 states that $\Pi_{ra}^m(w_a)$ is increasing on $(\tau(w), w]$ and maximized at w when $k \leq \underline{k}$. Therefore, $\Pi_{ra}^m(w_a)$ is maximized at w . \square

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