

e - c o m p a n i o n

ONLY AVAILABLE IN ELECTRONIC FORM

Electronic Companion—“Promised Lead-Time Contracts Under
Asymmetric Information” by Holly Lutze and Özalp Özer,
Operations Research, DOI 10.1287/opre.1080.0514.

An On-line Addendum to: “Promised Lead Time Contracts Under Asymmetric Information”

Holly Lutz^{*} and Özalp Özer[†]

Submitted: March 31, 2004

Proof of Proposition 13. Let p_L and p_H denote low and high penalty cost, respectively. Note that under Normally distributed demand, $G_s^*(\tau) = \alpha_s \sqrt{L+1-\tau}$ and $G_r^*(p_i, \tau) = \alpha_i \sqrt{l+1+\tau}$ where $\alpha_s = (h_s + p_s) \phi \left(\Phi^{-1} \left(\frac{p_s}{h_s + p_s} \right) \right) \sigma$ and $\alpha_i = (h_r + p_i) \phi \left(\Phi^{-1} \left(\frac{p_i}{h_r + p_i} \right) \right) \sigma$ for $i = \{L, H\}$. From Proposition 4 in Lutz and Özer (2004), the supplier’s optimal expected cost is

$$\lambda_L [G_s^*(\tau_L^a) - K_L^a] + (1 - \lambda_L) [G_s^*(\tau_H^a) - K_H^a] = \lambda_L \left\{ \begin{array}{l} G_s^*(\tau_L^a) + G_r^*(p_L, \tau_L^a) \\ + [G_r^*(p_H, \tau_H^a) - G_r^*(p_L, \tau_H^a)] \end{array} \right\} + (1 - \lambda_L) [G_s^*(\tau_H^a) + G_r^*(p_H, \tau_H^a)] - U_r^{max}.$$

For the rest of the proof we drop the constant U_r^{max} from the optimal cost. Since demand is Normally distributed (Propositions 11 and 12 in Lutz and Özer 2004), the optimal menu of contracts corresponds to the one yielding the smallest optimal expected cost for the supplier among the following.

| Contract menu: | Supplier expected cost: | |
|--------------------------------|--|-----|
| $\tau_L^a = \tau_H^a = 0$ | $\alpha_s \sqrt{L+1} + \alpha_H \sqrt{l+1}$ | (1) |
| $\tau_L^a = L+1, \tau_H^a = 0$ | $(1 - \lambda_L) \alpha_s \sqrt{L+1} + \alpha_H \sqrt{l+1} + \lambda_L \alpha_L (\sqrt{L+l+2} - \sqrt{l+1})$ | |
| $\tau_L^a = \tau_H^a = L+1$ | $\alpha_H \sqrt{L+l+2}$ | |

From Proposition 1(b) in Lutz and Özer (2004), we have $\alpha_H > \alpha_L$. Hence, we have the following.

$$\frac{(1 - \lambda_L) \alpha_s}{\alpha_H - \lambda_L \alpha_L} < \frac{\alpha_s}{\alpha_H} < \frac{\alpha_s}{\alpha_L}.$$

By comparing the expected costs in (1), we determine the optimal promised lead time. Let $\omega(L) \equiv [\sqrt{L+l+2} - \sqrt{l+1}] / \sqrt{L+1}$, we note that the optimal menu is $\tau_L^a = \tau_H^a = L+1$ when

$$\omega(L) \leq \frac{(1 - \lambda_L) \alpha_s}{\alpha_H - \lambda_L \alpha_L}, \tag{2}$$

^{*}School of Management, University of Texas at Dallas, Richardson, Texas 93085-0688, e-mail: hlutz@utdallas.edu

[†]Management Science and Engineering, Stanford University, Stanford, California 94305-4026, e-mail: oozer@stanford.edu

is $\tau_L^a = L + 1$ and $\tau_H^a = 0$ when

$$\frac{(1 - \lambda_L)\alpha_s}{\alpha_H - \lambda_L\alpha_L} < \omega(L) \leq \frac{\alpha_s}{\alpha_L}, \quad (3)$$

and is $\tau_L^a = \tau_H^a = 0$ when

$$\frac{\alpha_s}{\alpha_L} < \omega(L). \quad (4)$$

When we reduce the lead time L by one while keeping $L + l$ constant, we use $\omega(L - 1) = [\sqrt{L + l + 2} - \sqrt{l + 2}]/\sqrt{L}$ instead of $\omega(L)$. We have

$$\omega(L - 1) < \omega(L) < (\sqrt{l + 2} - \sqrt{l + 1})/(\sqrt{L + 1} - \sqrt{L}).$$

Next we show that the supplier with lead times L and l has a lower optimal expected cost than the supplier with $L - 1$ and $l + 1$. To do so, we consider three cases.

Case 1: Suppose $\omega(L - 1)$ satisfies (2), that is $\omega(L - 1) \leq \frac{(1 - \lambda_L)\alpha_s}{\alpha_H - \lambda_L\alpha_L}$. Because $\omega(L) > \omega(L - 1)$, $\omega(L)$ may satisfy (2), (3), or (4). If $\omega(L)$ satisfies (2), then from (1), the supplier's expected cost is $\alpha_H\sqrt{L + l + 2}$ both for lead times $L - 1$ and $l + 1$ and for L and l . If $\omega(L)$ satisfies (3), then the supplier's cost is higher for $L - 1$ and $l + 1$ when

$$(1 - \lambda_L)\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1} + \lambda_L\alpha_L(\sqrt{L + l + 2} - \sqrt{l + 1}) < \alpha_H\sqrt{L + l + 2}.$$

But this reduces to $(1 - \lambda_L)\alpha_s/(\alpha_H - \lambda_L\alpha_L) < \omega(L)$, which is true. If $\omega(L)$ satisfies (4), then the supplier's cost is higher for $L - 1$ and $l + 1$ when $\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1} < \alpha_H\sqrt{L + l + 2}$. This inequality reduces to $\alpha_s/\alpha_H < \omega(L)$, which is true because $\alpha_H > \alpha_L$.

Case 2: Suppose $\omega(L - 1)$ satisfies (3), so $\omega(L)$ may satisfy (3) or (4). If $\omega(L)$ satisfies (3), then from (1) the supplier's expected cost is higher for $L - 1$ and $l + 1$ when

$$\begin{aligned} \lambda_L\alpha_L(\sqrt{L + l + 2} - \sqrt{l + 1}) &< \lambda_L\alpha_L(\sqrt{L + l + 2} - \sqrt{l + 2}) \\ + (1 - \lambda_L)\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1} &< + (1 - \lambda_L)\alpha_s\sqrt{L} + \alpha_H\sqrt{l + 2}. \end{aligned}$$

We assume for a contradiction that the opposite is true, which yields the following.

$$\frac{(1 - \lambda_L)\alpha_s}{\alpha_H - \lambda_L\alpha_L} \geq \frac{\sqrt{l + 2} - \sqrt{l + 1}}{\sqrt{L + 1} - \sqrt{L}}$$

However, this contradicts the implication of $\omega(L) < (\sqrt{l + 2} - \sqrt{l + 1})/(\sqrt{L + 1} - \sqrt{L})$ and (3). If $\omega(L)$ satisfies (4), then the supplier's expected cost is higher for $L - 1$ and $l + 1$ when

$$\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1} < \lambda_L\alpha_L(\sqrt{L + l + 2} - \sqrt{l + 2}) + (1 - \lambda_L)\alpha_s\sqrt{L} + \alpha_H\sqrt{l + 2}.$$

Note that when $\omega(L)$ satisfies (4), we also have

$$\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1} \leq \lambda_L\alpha_L(\sqrt{L + l + 2} - \sqrt{l + 1}) + (1 - \lambda_L)\alpha_s\sqrt{L + 1} + \alpha_H\sqrt{l + 1},$$

so the strict inequality holds.

Case 3: Suppose $\omega(L-1)$ satisfies (4). From (1) the supplier's expected cost is higher for $L-1$ and $l+1$ when $\alpha_s\sqrt{L+1} + \alpha_H\sqrt{l+1} < \alpha_s\sqrt{L} + \alpha_H\sqrt{l+2}$. But this inequality reduces to

$$\frac{\alpha_s}{\alpha_H} < \frac{\sqrt{l+2} - \sqrt{l+1}}{\sqrt{L+1} - \sqrt{L}},$$

which is true because (4) is satisfied, $\omega(L) < (\sqrt{l+2} - \sqrt{l+1})/(\sqrt{L+1} - \sqrt{L})$, and $\alpha_H > \alpha_L$. \square

References

- [1] Lutze, H. and Ö. Özer. 2004. Promised Lead Time Contracts Under Asymmetric Information. To appear in *Operations Research*.