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Managing a product family under stochastic technological changes

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ABSTRACT

This paper characterizes a decision framework by which a firm can manage generational product replacements under stochastic technological changes. First, we characterize an optimal threshold-based product replacement policy that maximizes the firm's expected total profit for a finite planning horizon. With this policy, the firm should optimally replace its current product when the performance gap of the product is above a threshold. Upon each product replacement, we also show that the firm should adopt the latest technology for the new product. Second, using stochastic ordering concepts, we quantify the negative impact from the accelerated technological changes on the expected total profit. Finally, we provide an analytical example based on a consumer choice model and characterize a technology follower's joint product replacement and pricing decisions upon the arrival of each innovation from a technology leader.

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1. Introduction

Today's high-technology firms face increasing challenges in managing their product offerings to accommodate new technologies that evolve faster than ever before. Replacing products promptly according to technological changes is a key factor in establishing and preserving competitive status in the market. For example, during the 1980s and 1990s, Sony dominated the market for portable music players with its Walkman product line. Sony's capability to promptly adopt new technologies to improve its product offering was a major factor in being the market leader (Sanderson and Uzumeri, 1995). However, recently Sony failed to recognize and respond to technological advancements in hard disk and flash memory. The firm's slow response gave Apple Computers the opportunity to take over market dominance through a stream of successful iPod products that closely followed technolo-

gical trends (Edwards, 2005). Yet, such frequent product replacements could be very costly. In late 1980s, the computer industry boasted a fast pace of technology evolution and suffered from a significant profit decline (Lewis, 1989). Therefore, firms in a fast-changing industry must manage their product families to avoid the risk of innovating themselves out of business. In this paper, we consider a firm that manages a product family. The firm needs to decide whether and when to adopt a newer technology for its product so as to maximize total expected profits. Our model focuses on the scenario in which products differ in their technology content and replace one another in a serial fashion, a category that Uzumeri and Sanderson (1995) refer to as a generational product family. We analyze the product family problem with respect to two inter-connected aspects: planning product replacements under technology evolution and managing the product family for technology evolution.

1.1. Planning product replacements

Planning product replacements under technology evolution is a process of deciding when to replace an old

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product with a new one and which new technology to adopt for the new product. Firms follow different product replacement strategies even for products that use similar technologies. Consider, for example, the computer and the game console industries. Both computers and game consoles rely heavily on technologies such as the micro-processor. While computer makers are keeping close pace with technological changes in the microprocessor, game console manufacturers tend to bundle technological improvements and introduce new products over a much longer time interval.

We model the product replacement process as a Markov decision model in which technology evolution follows a Markov process that captures the uncertainties in both the timing and the content of technological changes. We establish the optimality of a threshold-type policy with which a firm can optimally replace its products according to technological changes. In particular, the firm should replace its product when the product's performance gap with respect to the latest available technology is above a certain threshold. In addition, we show that upon each product replacement, the new product should always adopt the latest technology rather than adopting an intermediate technology, that is, it is never optimal to adopt an intermediate technology.

The literature in technology adoption (see, for example, Balcer and Lippman, 1984; Kornish, 1999; Krankel et al., 2006; Paulson Gjerde et al., 2002) and machine replacement (see, for example, Derman, 1963; Nair, 1995) document threshold-based policies for their respective problems. One popular approach to capture the benefits from adopting an improved technology has been to assume that a product's profit is strictly increasing in the performance of its technology content. However, in certain industries, such as the computer industry, this approach overstates the profitability of newer technologies, hence biasing decisions toward chasing technological trends. First, this approach does not assume any limit on a product's profitability due to market competition and consumer preference. Though technological performance can advance without a boundary, a product's profitability is limited by the content of its competing peers as well as consumers' preference on technological performance. Second, this approach does not take into consideration the cost of adopting improved technology. Normally, a new technology itself comes with a cost premium, which can offset the price premium of a product that adopts the new technology.

We contribute to this literature by using a profit model in which a product's profit is decreasing in its *performance gap*—the difference between the performance of the product's technological content and that of the latest technology. The benefits of using performance gap as a proxy for profit are three-fold. First, this setup captures the technological competitiveness from adopting an improved technology while reflecting the limit on profitability due to market competition. Second, specifying product profit with the Multinomial Logit model, we assure that our results fit the market segment where (1) technologies with low performance have a small cost advantage; (2) customers place a high weight on product

performance; and (3) consumers have low price sensitivity. Third, this profit model enables us to capture the negative consequences of accelerated technology evolution and motivates us to extend our analysis beyond planning product replacement and also consider how to manage a product family.

1.2. Managing a product family for technology evolution

Managing a product family for technology evolution is the process of seeking the best strategy for a firm to maintain its financial strength in the case of accelerated technological changes. Rosenberg (1976) and Balcer and Lippman (1984) argue that firms have several incentives to wait for forthcoming technologies, hence to postpone product replacement. Our results, however, show that this argument is not always true. The attractiveness of postponing product replacement depends on the characteristics of a firm's revenue function. By postponing product replacement, a firm can benefit from two sources: (1) smaller product replacement cost due to possibly fewer product replacements and (2) higher revenue from a forthcoming technology. The firm, however, has to sell an inferior product in the market for at least one more period, which usually incurs revenue losses. This trade-off does not always justify postponing product replacement.

Another popular idea in many fast-changing industries is that a firm should follow technological trends closely and reduce its product replacement thresholds to keep its competitive status. We demonstrate that a firm needs to replace its products more frequently as technology evolution accelerates. But, we also show that having more product replacements is not equivalent to having lower product replacement thresholds. A firm that blurs the difference between these two concepts could reduce its product replacement thresholds blindly, hence diverging from the optimal policy and experiencing diminishing profits together with shortened product lifecycles.

One important question in managing a product family is to determine whether implementing an optimal product replacement policy alone is enough to maintain a firm's financial strength as technology evolution speeds up. We show that the firm's total expected profit under the optimal threshold-based product replacement policy declines as the pace of technology evolution accelerates. Hence, firms in fast-changing industries are under the risk of innovating themselves out of business and should seek additional options to mitigate the negative impact from fast technology evolution.

Our results show that reducing product replacement cost is an effective approach in dealing with accelerated technology evolution. With a lower product replacement cost, a firm reaps a two-fold benefit: spending less on each product replacement and keeping closer pace with technological trends by speeding up product replacement. In the extreme, when a firm's product replacement cost is sufficiently low, the pace of technology evolution will not have any impact on the firm's profitability. This result explains the difference in practice between the PC industry and the game console industry. For the past

decade, the computer industry has continuously pushed down product replacement cost through initiatives such as open-architecture and modular design. Compared with the PC industry, product replacement cost in the game console industry is much higher because game console manufacturers need to spend a large amount of resources to encourage game development firms to provide contents for new game consoles (Schilling, 2003). As a result, product replacements in the PC industry are more frequent than those in the game console industry.

We provide additional results and insights with an analytical model that captures well, for example, the situation in PC industry in the 1980s when technological advancements were originating from leaders, such as Apple Computer and IBM. In particular, we develop a model in which innovations from a technology leader drive technology evolution, and a technology follower needs to make a product replacement decision upon the arrival of each innovation. With the help of the analytical model, we determine optimal product replacement and pricing decisions for the follower. We also screen out competitive strategies for both the technology leader and the follower. To fence off the technology follower, we determine that the leader could either speed up its innovation process or raise the follower's product replacement cost with barriers like copyright and patent protection. To maintain its profitability, the technology follower should emphasize initiatives that reduce its product replacement cost.

Technology evolution has been considered in many decision problems, such as in product replacement problems (Paulson Gjerde et al., 2002; Krankel et al., 2006), machine replacement problems (Nair, 1995), and production process switching problems (Chambers, 2004; Chambers and Kouvelis, 2003). However, the impact of technology evolution on a firm's profitability is rarely considered. An exception is von Braun (1990, 1991) who explains that the shortening of product lifecycle induced by continuous technology innovation could result in a boom-bust pattern in a firm's profitability. His study, however, depends largely on demand analysis and does not consider the product replacement decision at all. Our study contributes to this literature by characterizing the negative impact from the accelerated pace in technology evolution and justifying the financial benefits of industrial initiatives in reducing product replacement cost.

The product replacement cost for a product family depends closely on the family's architecture and production process, factors that are usually the outcomes of the family's development process. Models of product development usually concentrate on a single product development process and address how a new technology can be brought to market efficiently. The trade-off is postponing product launch to improve the quality of the product, versus introducing early to earn profits sooner (see, for example, Cohen et al. 1996a, b and references therein). Our results show that product replacement cost is another important factor that should be considered in product development models, especially for product families that are experiencing rapid technology evolution. Technology also plays an important role in product line positioning

problems in which a family of products can coexist in a market. Studies in this group often focus on analyzing the cannibalization effect among a set of products and try to determine the optimal sequence and/or scope of product offerings. We refer interested readers to Krishnan et al. (1999) as well as Netessine and Taylor (2005) for more details on the dynamics between technology and product line positioning.

The rest of the paper is organized as follows. In Section 2, we formulate the decision model based on a general single-stage profit function and characterize an optimal product replacement policy for a finite planning horizon. In Section 3, we determine the effect of accelerated technology evolution and discuss strategies to manage product families to cope with accelerated technology evolution. In Section 4, we provide an explicit single-stage profit function using a consumer choice model based on a Multinomial Logit model. Using this model, we provide additional results and managerial insights. In Section 5, we summarize and conclude the paper.

2. Planning product replacements

We study a firm that plans its product replacements given an exogenous, stochastically evolving technology process. This process is such that the arrival time and the performance advancement of each new technology are uncertain. As a technology taker, the firm develops its products based on available technologies and must schedule its product offerings according to its knowledge of a future technology path. In addition, the firm offers at most one product in each stage and targets its product offerings to the same customer segment. We formulate this problem and characterize the firm's two key decisions: (1) when to replace an old product and (2) which technology to adopt in the new product offering. As an example to this decision process, consider a personal computer assembler that incorporates off-the-shelf technology, such as a CPU, into its products. The assembler needs to plan its product offerings based on the evolution of CPU technologies provided by a semiconductor manufacturer, such as Intel.

2.1. The product replacement model

We model technology evolution as a continuous-time Markov process. Technology evolution generates a sequence of new technologies that provide higher performance levels. Over time a new technology emerges, but the arrival time and performance level of this new technology are uncertain. This process can be modeled by a discrete-time Markov process describing the transition probabilities among technology states and an independent time homogenous Poisson process describing the arrival of new technology. We denote the Markov process as (X, λ) , where X represents the embedded discrete-time Markov process and $\lambda > 0$ is the parameter for the exponentially distributed interarrival time. The process X has homogeneous transition probability. Let $Q = \{q_z\}$ be the transition matrix of X , where $q_z \in [0, 1]$

represents the probability that upon a transition technology performance will increase by $z \in Z$. And Z is a finite set denoting positive increments (integers or real numbers) representing performance improvement at a given stage, i.e., the time interval between two successive technology arrivals.

We assume that a product's profit rate $R(v) > 0$ is a decreasing function of v , the performance gap between the product and the latest technology. This assumption reflects both the benefits of adopting a new technology and the natural limit of such a benefit. First, it shows that a product's profit rate is higher when its performance is closer to the latest technology. Second, the arrival of a new technology reduces a product's profit rate since it widens the product's performance gap. Third, products with the same performance gap have the same profit rate, regardless of their absolute technology performance. Finally, the profit rate is bounded by $R(0) < \infty$.

We consider a product replacement planning problem with n stages. Suppose that at the beginning of stage $i \in [1, n]$ the firm's product has a performance gap v_i . Taking future technology evolution into consideration, for the product offered in stage i the firm chooses the desired performance gap $v \in P_i$, where the set P_i depends on existing technologies that the manager can use to reduce the performance gap. To construct this decision set, the manager needs to keep track of available technologies or the performance gap between the latest technology and all other existing technologies. Hence, the state space is given by (P_i, v_i) . If $v \neq v_i$, then a product replacement is performed at a fixed cost K_F . Demand realizes during stage i at a profit rate $R(v) \in (0, \infty)$. At the end of stage i , a new technology arrives with performance improvement of $z_i \in Z$. This arrival widens the performance gap of the existing product and also affects the decision set available for the next stage. Hence, the state space is updated as

$$v_{i+1} = v + z_i, \\ P_{i+1} = \{p | p + z_i, \forall p \in P_i\} \cup \{0\},$$

where $P_0 = \{\emptyset\}$.

Let $J_i(P_i, v_i)$ denote the maximal total expected profit from stage i until the end of stage n . Under a continuous discount rate α , the expected total profit from stage i is

$$\int_0^\infty \lambda e^{-\lambda t} \left(\int_0^t e^{-\alpha s} R(v) ds \right) dt = \frac{1}{\lambda + \alpha} R(v),$$

where v is the performance gap for the product offered in stage i . The discounted value of the profit-to-go function is

$$\int_0^\infty \lambda e^{-\lambda t} e^{-\alpha t} J_{i+1}(P_{i+1}, v + z_i) dt = \frac{\lambda}{\lambda + \alpha} J_{i+1}(P_{i+1}, v + z_i).$$

Let β_1 and β_2 denote, respectively, $1/(\lambda + \alpha)$ and $\lambda/(\lambda + \alpha)$. We can formulate the dynamic program for the product replacement problem as

$$J_i(P_i, v_i) = \max_{v \in P_i} \left\{ -\delta(v - v_i)K_F + \beta_1 R(v) + \beta_2 \sum_{z_i \in Z} q_{z_i} J_{i+1}(P_{i+1}, v + z_i) \right\}, \quad (1)$$

where $\delta(v - v_i) = 0$ if $v = v_i$ and $\delta(v - v_i) = 1$ otherwise. At the end of stage n , the terminal value of this dynamic program is

$$J_{n+1}(\cdot, v_{n+1}) \equiv H(v_{n+1}). \quad (2)$$

One interpretation of the terminal value is that $H(\cdot)$ represents the firm's expected future revenues from continuing to sell the product offered in stage n . We assume that $H(v_{n+1})$ is decreasing in v_{n+1} .

2.2. An optimal product replacement policy

Since the maximum profit at each stage is bounded from above by $R(0)$, the n -stage dynamic problem in Eq. (1) has an optimal solution. The following proposition provides a starting point to characterize an optimal product replacement policy.

Proposition 1. Let $V_n^* = \{v_1^*, \dots, v_n^*\}$ denote an optimal product replacement sequence for a sample path $\{z_1, \dots, z_n\}$ of the n -stage dynamic program, where v_i^* is the optimal performance gap for product in stage i and v_i is the product's performance gap at the beginning of stage i . Then (1) $v_i^* \leq v_i$ and (2) $v_i^* = 0$ if $v_i^* \neq v_i$.

Proposition 1 helps reduce the complexity of the decision problem significantly. With part (1) of the proposition, at stage i , rather than evaluating each option in P_i , we can limit the search for optimal action to $\{v : v \in P_i, v \leq v_i\}$. That is, the firm shall not reduce its product's technology performance as technology evolves. With part (2), we can further reduce this search to $\{0, v_i\}$. As a result, instead of keeping track of all available technologies, at each stage i the firm needs to compare only two options: keeping the current product ($v = v_i$) or replacing it with a new product that adopts the latest available technology ($v = 0$). Given this state-space reduction result, we can simplify the dynamic program in Eq. (1) as

$$J_i(v_i) = \max_{v \in \{0, v_i\}} \left\{ -\delta(v_i - v)K_F + \beta_1 R(v) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v + z) \right\} \\ = \max \left\{ \beta_1 R(v_i) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v_i + z), \right. \\ \left. -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(z) \right\}. \quad (3)$$

For the rest of the paper, we drop the subscript i from the technology performance improvement z because the technology process has homogeneous transition probabilities. To facilitate our analysis, we introduce the following definitions:

$$W_i^k(v_i) \equiv \beta_1 R(v_i) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v_i + z), \quad (4)$$

$$W_i^r \equiv -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(z). \quad (5)$$

Notice that $W_i^k(v_i)$ represents the total expected profit if the firm keeps the current product at least one more stage, and W_i^r is the total expected profit if the firm immediately

replaces the current product with a new product that closes the performance gap.

Using $W_i^k(v_i)$ and W_i^r , the dynamic program in Eq. (3) is equivalent to

$$J_i(v_i) = \max\{W_i^k(v_i), W_i^r\}$$

$$= \max\{0, W_i^r - W_i^k(v_i)\} + W_i^k(v_i).$$

Let $W_i(v_i) \equiv W_i^r - W_i^k(v_i)$ denote the expected net profit from replacing a product at the beginning of stage i . Then we have

$$J_i(v_i) = \max\{0, W_i(v_i)\} + W_i^k(v_i). \quad (6)$$

From Eq. (6), if $W_i(v_i) > 0$, then it is optimal to replace the current product and close the performance gap; otherwise, it is optimal to keep the current product. Hence, the product replacement decision at stage i is determined by the sign of $W_i(v_i)$.

We define the threshold \bar{v}_i as

$$\bar{v}_i \equiv \begin{cases} \min\{v : W_i(v) \geq 0, v \geq 0\} & \text{if } \max_{v \in [0, v_i]} W_i(v) \geq 0 \\ \infty & \text{otherwise.} \end{cases} \quad (7)$$

The following proposition characterizes an optimal product replacement policy.

Proposition 2. *The following results are true for stage $i \in [1, n]$:*

- (1) $W_i^k(v_i)$ is decreasing in v_i .
- (2) W_i^r is independent of v_i .
- (3) $W_i(v_i)$ is increasing in v_i .
- (4) $J_i(v_i)$ is decreasing in v_i .
- (5) An optimal policy is $v_i^* = 0$ if $v_i \geq \bar{v}_i$ and $v_i^* = v_i$ otherwise. The threshold \bar{v}_i is defined in Eq. (7).

Parts (1) and (3) of Proposition 2 state that when the performance gap of the current product is larger, the expected profit from keeping the product is smaller and the benefit from replacing it is larger. Part (4) implies that the optimal expected profit is decreasing in the performance gap of the current product. Part (5) of Proposition 2 shows that the optimal product replacement policy has a threshold structure. That is, if the performance gap of the current product is below that stage's performance threshold, it is optimal not to replace the current product. Otherwise, it is optimal to replace the current product with the new one that adopts the latest available technology and closes the performance gap to zero.

3. Managing a product family for technological evolution

We have shown with Proposition 2 that a threshold-based strategy for product replacements under technological changes is optimal. This strategy, however, is passive in the sense that the path of technological changes determines the path of product replacements. In other words, given a fixed product replacement cost, the firm's optimal total profit depends on the property of the

technology evolution process. As we will show in this section, when the pace of technology evolution accelerates, the firm's optimal expected profit will decrease. Hence following an optimal product replacement strategy may trap the firm to also follow a faster pace technology adoption process despite facing lower profits.

3.1. The impact of technology evolution

We analyze how changes in the pace of technology evolution can affect the firm's profitability. Recall that we model technology evolution as (X, λ) , where X denotes a discrete-time Markov process describing the transition probabilities among states and λ is the parameter of the exponentially distributed sojourn time at each state. We compare product replacement decisions and resulting profits under the process (X, λ) to those under a new process (X', λ) . We assume that X and X' have the same transition set Z but different transition probabilities.

Let X_s and X'_s denote, respectively, the states that X and X' will visit from state $s \in I$. We model the relationship between X_s and X'_s with the *stochastically larger* concept (Ross, 1996).

Definition 1. The random variable X is stochastically larger than the random variable Y , written $X \geq_{st} Y$, if $P\{X > a\} \geq P\{Y > a\}$ for all a .

An important property of the *stochastically larger* relationship is that, for all increasing functions $f(\cdot)$, $E[f(X)] \geq E[f(Y)]$ if $X \geq_{st} Y$.

To facilitate our analysis, we define $Z_u \equiv \{z | z \leq u, z \in Z \text{ and } u \in Z\}$ as the subset of the transition set Z for a given $u \in Z$.

Proposition 3. $X_s \geq_{st} X'_s$ if and only if $\sum_{z \in Z_u} q_z \leq \sum_{z \in Z_u} q'_z$ for all $u \in Z$.

Proposition 3 shows that process X is more likely to make larger jumps than process X' . If $X_s \geq_{st} X'_s$ for every $s \in I$, we say that the pace of X is *stochastically faster* than that of X' . Notice that $\sum_{z \in Z_u} q_z \leq \sum_{z \in Z_u} q'_z$ for all $u \in Z$ also implies that the underlying process for (X, λ) is stochastically larger than that for (X', λ) (see Stoyan, 1983, p. 68).

Let $J_i(v_i)$ and $J'_i(v_i)$ denote, respectively, the optimal expected total profit under X and X' by following the threshold-base policy presented in Proposition 2.

Proposition 4. If $X \geq_{st} X'$, then we have $J_i(v_i) \leq J'_i(v_i)$.

Proposition 4 shows that the expected total profit is higher when the pace of technology evolution is slower. Recall that under a slower technology evolution process the probability of having a large performance improvement is smaller. We normally expect a lower profit margin for products with smaller performance improvement in technology, which may appear to contradict Proposition 4. Careful examination, however, reveals that Proposition 4 is valid because a product's profit rate depends on its performance gap but not on its technology performance alone. When technology is evolving faster, the performance gap of an existing product is increasing faster,

hence the product's profit rate is decreasing faster. In addition, a product replacement with a larger performance improvement does not bring a higher profit rate, because the maximum profit rate is the same for all products with a zero performance gap.

Proposition 4 implies that, when technology evolution accelerates, the firm may not maintain its financial strength by simply following an optimal product replacement policy. Hence, the firm has to seek other strategies to mitigate the negative impact from the accelerated pace of technology evolution.

3.2. The trap of following the technology trend

When technology evolution accelerates, a common response is to reduce product replacement thresholds so that the firm can replace products more frequently to mitigate the negative impact from larger technological jumps. This strategy, however, could trap the firm into a situation where diminishing profit accompanies shortened product lifecycles.

First of all, lowering the product replacement threshold does not change anything in the firm's cost structure, hence it cannot achieve a higher total profit than the threshold-based strategy does. In other words, replacing product more frequently will shorten the product lifecycle without improving the firm's overall profitability. In addition, simply lowering product replacement thresholds can result in worse profitability since a faster pace of technology evolution does not always result in a lower product replacement threshold.

Proposition 5. Let \bar{v}_n and \bar{v}'_n denote, respectively, the product replacement thresholds under technology process X and X' at stage n . If the terminal value $H(\cdot)$ is convex, then we have $\bar{v}_n \geq \bar{v}'_n$; if $H(\cdot)$ is concave, then we have $\bar{v}_n \leq \bar{v}'_n$.

Proposition 5 states that at stage n the product replacement threshold under a slower technology evolution process could be either higher or lower than that under a faster one. Recall that a product replacement always closes the performance gap to zero. Hence, regardless of the technology process X or X' , a product replacement at the last stage n brings the same profit gain in stage n . Therefore, the product replacement thresholds are determined by the expected terminal benefits after stage n , which are $\sum_z q_z(H(z) - H(v_n + z))$ and $\sum_z q'_z(H(z) - H(z + v_n))$ under process X and X' respectively. Since X is stochastically larger than X' , $\sum_z q_z(H(z) - H(z + v_n))$ will be larger than $\sum_z q'_z(H(z) - H(z + v_n))$ if $H(z) - H(z + v_n)$ is increasing in z . The convexity or concavity of the terminal function $H(\cdot)$ determines whether we will see an increasing or decreasing pattern in the terminal benefit $H(z) - H(z + v_n)$. Notice that we can apply similar arguments to other stages as well. Hence, when the pace of technology evolution accelerates, simply lowering the product replacement threshold can easily break the optimality of the threshold-based replacement policy and put the firm into financial difficulties.

3.3. Managing product replacement cost for technology evolution

As the pace of technology evolution accelerates, in addition to following optimal product replacement policies, the firm has to seek other approaches to avoid deterioration in its financial position. For example, in the computer and electronics industries, firms take initiatives such as modular and platform-based design to reduce their product replacement costs. The benefits from reducing product replacement cost are two-fold. First, the cost for each product replacement is lower. Second, a lower replacement cost triggers more product replacements, hence leading to higher profit rates from the product family. However, reducing product replacement cost itself is costly since it usually involves more sophisticated design in a product's physical architecture and production process. Hence, understanding the benefits from reducing product replacement cost is essential to leverage it as a tool to break the trap of following technology trends.

Proposition 6. For $X \geq_{st} X'$, if $K_F \leq \beta_1(R(0) - R(\underline{z}))$ where $\underline{z} \equiv \min\{z : z \in Z\}$ and $\sum_{z \in Z} q_z H(z) = \sum_{z \in Z} q'_z H(z)$ for all $z \in Z$, then we have $J_i(v_i) = J'_i(v'_i)$ for all $v_i, v'_i \geq 0$.

Proposition 6 shows that the pace of technology evolution has no impact on the expected total profit at all when the product replacement cost is sufficiently low. The underlying reasons are three-fold. First, the product replacement cost is low enough so that a product replacement is always optimal at each stage. Second, a product's profit rate depends on its performance gap only, thus the profit rate is the same for all products with a zero performance gap. Third, the expected terminal value at the end of the planning horizon does not depend on the underlying technology evolution process. Because of the first two reasons, no matter what the pace of the underlying technology evolution process is, at each stage we have a product replacement and obtain the same profit rate from the new product. Hence, the expected total profit does not depend on the pace of technology evolution.

Proposition 6 confirms that reducing product replacement cost is an effective approach to mitigate the impact of accelerated technology advancement. Ideally, firms should strive to reduce their product replacement costs as low as possible so that they can replace their product offerings as soon as technology changes. In reality, however, reducing product replacement cost is very costly and sometimes not feasible to achieve.

Industrial practices from the computer and the game console industries demonstrate how the difference in product replacement costs has an impact on the product replacement patterns. Products from both industries rely on technologies such as the microprocessor. Initiatives like modular and platform-based design can significantly reduce the cost associated with replacing hardware. Compared with the computer makers, game console manufacturers are experiencing high cost associated with product replacements since they depend heavily on

outside game-developing firms to provide new software content for new game consoles (Schilling, 2003). As a result, while computer makers are keeping close pace with microprocessor technology changes, game console manufacturers tend to bundle technology improvements and introduce new products over a much longer time interval.

4. Analytical example with multinomial logit model

Here we provide additional results and insights through an analytical example in which a technology follower is competing with a technology leader. Innovations from the technology leader form the technology evolution process. As an innovator, the technology leader always offers a product with the highest technology performance at a fixed price. As a technology taker, the technology follower plans its product replacement decision based on the technology leader's innovations.

4.1. Optimality of the threshold policy

We analyze the technology follower's product replacement decision with the dynamic program in Eq. (6). To simplify our analysis, we assume zero salvage value at the end of the planning horizon; that is, $H(\cdot) \equiv 0$. Our results, however, still hold when $H(\cdot)$ is decreasing in the performance gap.

Suppose that at stage i the technology leader introduces a new product at price p_i^l . Observing the leader's new technology, the follower makes its product replacement decision and offers the product with the performance gap v at the price p_i . The performance gap here is the difference in technology performance between the technology leader's product and the technology follower's product at stage i .

The technology follower competes with the technology leader for customers that arrive at a rate D_i during stage i . Each arriving customer selects a product based on its utility. Let w_t and w_p represent, respectively, individual attribute weights on technology level and price. At stage i , a customer's expected utilities from the follower's and the leader's products are, respectively, $\mu - w_t v - w_p p_i$ and $\mu^l - w_p p_i^l$. Note that the leader's product has zero performance gap and sells at the price p_i^l . The constants μ and μ^l denote, respectively, an individual customer's base utility on the follower's product and the leader's product. Notice that, instead of using product performance as a positive utility, we use the performance gap as a negative utility. The implicit assumption is that a customer always weighs the performance of a product against the performance of the technology leader's product.

Let $N(v, p_i)$ represent the probability that a utility-maximizing customer will select the product with performance gap v and selling at price p_i . Using the Multinomial Logit (MNL) model to describe consumers' choices, we have

$$N(v, p_i) = \frac{e^{(\mu - w_t v - w_p p_i)}}{e^{(\mu - w_t v - w_p p_i)} + e^{(\mu^l - w_p p_i^l)}} \quad (8)$$

The MNL model is extensively used in marketing (Ofek and Srinivansan, 2002) and has attracted increasing attention from operations management (Aydin and Hausman, 2003). For a detailed discussion of the MNL model, see van Ryzin and Mahajan (1999).

A product's profit rate also depends on its production cost. We model the unit production cost of the product with performance gap v as

$$C(v) = C_F - \gamma v, \quad (9)$$

where C_F denotes the unit cost of choosing the latest technology and γv denotes savings in cost from choosing a technology other than the latest one. From Eq. (9), a product's unit cost is decreasing in its performance gap, reflecting the impact from technological obsolescence that is common for computer and electronic products.

Combining the product's cost, price, and demand rate, we have the technology follower's profit rate in stage i as

$$R(v, p_i) \equiv (p_i - C(v))N(v, p_i)D_i \\ = (p_i - (C_F - \gamma v)) \frac{e^{(\mu - w_t v - w_p p_i)}}{e^{(\mu - w_t v - w_p p_i)} + e^{(\mu^l - w_p p_i^l)}} D_i. \quad (10)$$

Eq. (10) indicates that a product's profit rate $R(v, p_i)$ is a function of its performance gap v and price p_i . Given a product with performance gap v , the technology follower should first optimize over the price p_i to obtain the optimal profit rate achievable for the product at stage i . We use $R(v) \equiv R(v, p_i^*(v))$ to denote the optimal profit rate for the follower's product, where $p_i^*(v) \equiv \operatorname{argmax}_{p_i} R(v, p_i)$ denotes the product's optimal price. Correspondingly, we use $N(v) \equiv N(v, p_i^*(v))$ to represent the probability that a customer will choose the technology follower's product selling at the price $p_i^*(v)$.

Proposition 7. For the profit rate function $R(v, p_i)$ given in Eq. (10), we have

- (1) Given v , there exists a unique $p_i^*(v)$ that satisfies $\partial R(v, p_i) / \partial p_i = 0$. Furthermore, $p_i^*(v)$ maximizes $R(v, p_i)$.
- (2) $p_i^*(v)$ is decreasing in v .
- (3) $N(v)$ is decreasing in v when $\gamma \leq w_t / w_p$ and is increasing in v otherwise.
- (4) $R(v)$ is decreasing in v when $\gamma \leq w_t / w_p$ and is increasing in v otherwise.

Parts (1) and (2) of Proposition 7 state that at each stage the technology follower can find a unique optimal price to maximize its profit rate and the optimal price is higher if its product has a smaller performance gap. Parts (3) and (4) of Proposition 7, however, show that the technology follower does not always benefit from a smaller product performance gap. Reducing the performance gap has a two-sided impact. On the one hand, from Eq. (8), reducing the performance gap v increases customers' utility. On the other hand, from part (1) of Proposition 7, reducing the performance gap increases a product's optimal price, which reduces customers' utility. Hence, a product with a smaller performance gap can have a lower probability of being chosen by customers. Additionally, from Eq. (9), a smaller performance gap also incurs a higher production cost. Combining the above effects, when the market's

value for a performance improvement is smaller than the marginal cost of that improvement, the technology follower will observe a lower profit rate.

With the results from Proposition 2, we characterize an optimal product replacement policy for the technology follower in the following proposition.

Proposition 8. *Suppose that product replacement cost is K_F and $R(v, p_i)$ is from Eq. (10). Let \bar{v}_i be defined as in Eq. (7). If $\gamma \leq w_t/w_p$, then an optimal policy is $v_i^* = 0$ if $v_i \leq \bar{v}_i$ and $v_i^* = v_i$ otherwise.*

Proposition 8 shows that using the threshold-based policy the technology follower will achieve optimal profit when $\gamma \leq w_t/w_p$. The condition $\gamma \leq w_t/w_p$ is not as stringent as it may appear. Notice that $\gamma = -dC(v)/dv$ represents the marginal cost of reducing the performance gap and

$$\frac{w_t}{w_p} = -\frac{\partial R(v, p_i)/\partial v}{\partial R(v, p_i)/\partial p_i}$$

represents the market's value for performance improvement. The condition $\gamma \leq w_t/w_p$ could be satisfied with a small γ , indicating that technologies with a high performance level have a small cost disadvantage; with a large w_t , indicating that customers put a large weight on a product's technology performance; or with a small w_p , indicating that customers are less sensitive to price. Quite often customers from high-end market segments put a high weight on product performance but a low weight on price, which indicates a large w_t/w_p .

4.2. Numerical examples

Through numerical examples we extract additional insights on the technology follower's problem in managing its product family. Consider a base case with the following parameter values. On the technology leader's side, the set of performance improvements is $Z = \{1, 2, 3, 4\}$ and the probabilities for technological transitions are $Q_0 = (0.2, 0.3, 0.3, 0.2)$. The leader's sojourn time at each stage is exponential with parameter $\lambda = 1$. In addition, the technology leader always offers products with zero performance gap and at a price of 32. The planning horizon for the base case has 10 stages. The continuous discount rate is $\alpha = 0.1$, and all products have zero salvage value at the end of the planning horizon.

Table 1 presents the other parameter values for the base case. Notice from the table that an individual customer gets higher reservation utility from the leader's product, which is justifiable as the impact from the leader's market image. Table 1 also shows that $\gamma \leq w_t/w_p$; that is, the marginal cost of reducing the performance gap is smaller than the market's value on performance improvement. According to Proposition 8, the technology follower can find an optimal threshold-based policy for its product replacement problem.

Observation 1. *The acceleration in the leader's innovation pace will suppress the follower's profitability and could result in a lose–lose situation.*

Table 1

Parameter values for the base case.

Parameters	C_F	γ	w_t	w_p	D_i	μ	μ^L	K_F
Value	20	0.1	1	1	50	60	66	220

Proposition 4 shows that the acceleration of technology evolution will have a negative impact on a firm's profitability. We gain more business insights in this direction by evaluating the product replacement decision model under two other transition matrices, $Q_1 = (0.7, 0.1, 0.1, 0.1)$ and $Q_2 = (0.1, 0.1, 0.1, 0.7)$. Based on the stochastic relationship specified in Proposition 3, Q_1 defines the leader's slowest innovation process and Q_2 defines the fastest one.

In Figs. 1 and 3, the solid curves plot the follower's expected total profit under different replacement costs for Q_0 , while the dotted curves and the dashed curves, respectively, do the same for Q_1 and Q_2 . Among the three curves in Figs. 1 and 3, the dotted curve under Q_1 is always on the top, and the dashed curve under Q_2 is always at the bottom, indicating that the acceleration in the leader's innovation pace suppresses the follower's profitability.

The curves in Figs. 2 and 4 plot the leader's expected total profit in the same pattern as the curves for the follower's expected profit. In both figures, the leader's expected profit does not always increase in the pace of his innovation process. When the follower's product replacement cost is low, the acceleration in the leader's innovation process could result in a lose–lose situation. For example, as shown in Figs. 1 and 2, when the follower's replacement cost is 100, both the follower's and the leader's expected profits drop as the leader speeds up his innovation process.

Observation 2. *Lowering the product replacement cost could give the follower a significant competitive advantage over the leader.*

As the follower's product replacement cost decreases, the gaps between each pair of curves in Figs. 1 and 3 also narrow, a pattern indicating that the impact from the leader's innovation pace on the follower's profit is diminishing as the follower's product replacement cost decreases. When the product replacement cost is low enough, all three curves merge into one point, indicating that the changes in the leader's innovation pace do not affect the follower's expected total profit any more.

The follower's product replacement cost has significant impact on both the follower's and the leader's profits. As shown in Figs. 2 and 4, the leader's profit is increasing in the follower's product replacement cost, especially when the leader can establish a barrier to fence off the follower. Figs. 1 and 3 illustrate that the follower's profit is decreasing in its product replacement cost and can actually exceed the leader's profit.

Observation 3. *The follower's product replacement threshold increases as the pace of the leader's innovation process accelerates.*

In Table 2, under each technology process, the left column lists the follower's product replacement thresh-

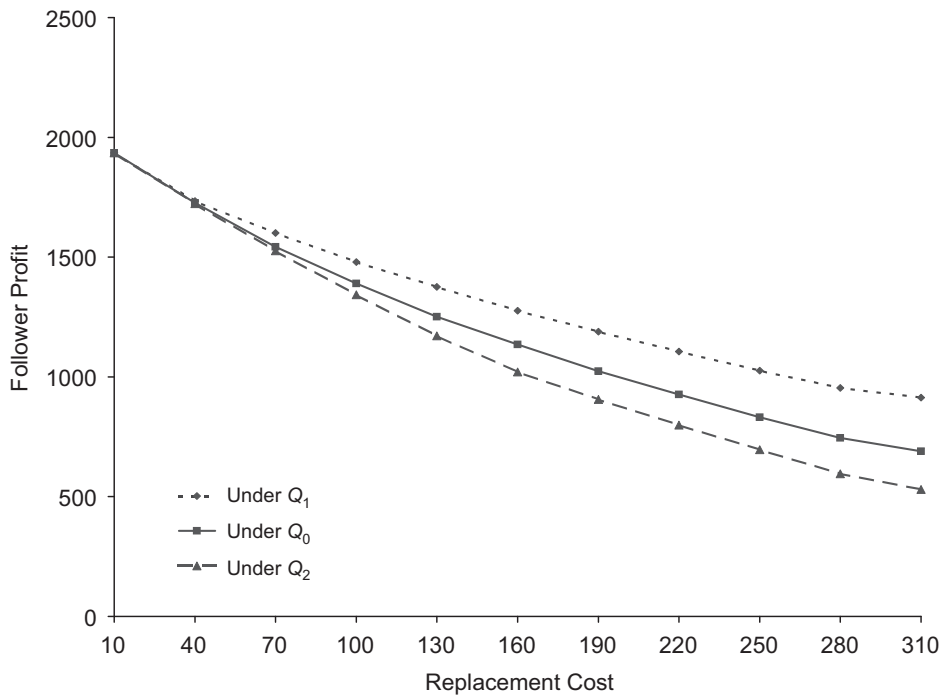


Fig. 1. The follower's expected profits when the leader sells her product at a price of 35.

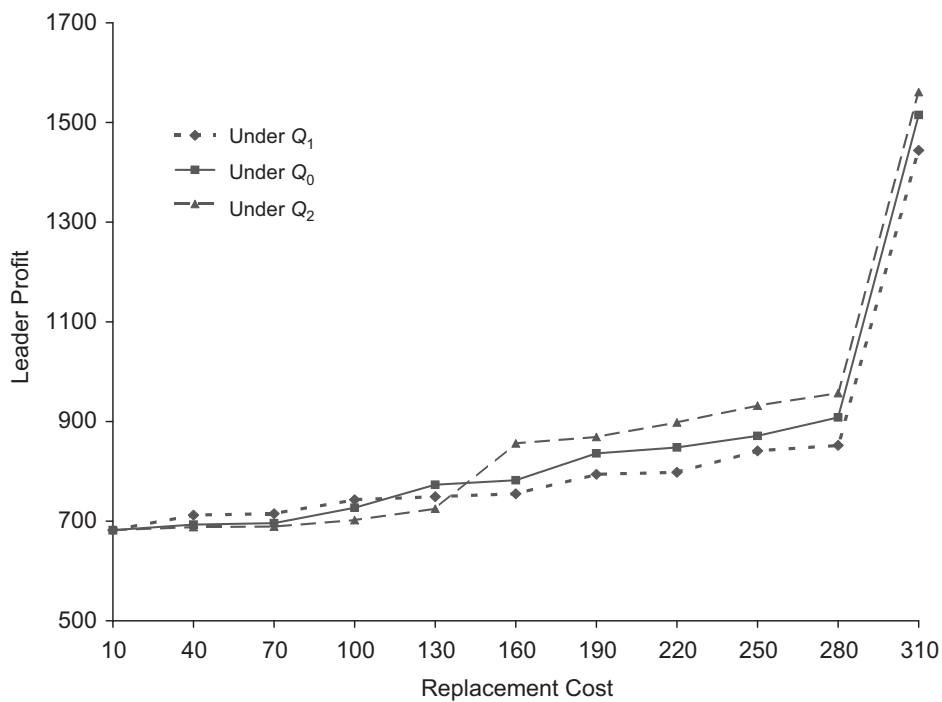


Fig. 2. The leader's expected profits when she sells her product at a price of 35.

olds at stage 4 for different product replacement costs, and the right column lists the expected length of product lifecycles $E[N]$ under the corresponding threshold on the left, where N denotes the number of stages that a product stays in the market. The first observation from Table 2 is that under each technology process the product replacement threshold is decreasing in the product replacement cost, indicating that a smaller product replacement cost

will always encourage the technology follower to have more product replacements.

As the pace of technology evolution accelerates, a common intuition is that the technology follower should replace its product more frequently so that she keeps closer pace with the technology leader. Since a lower product replacement threshold indicates a higher probability of product replacement, a natural response to the

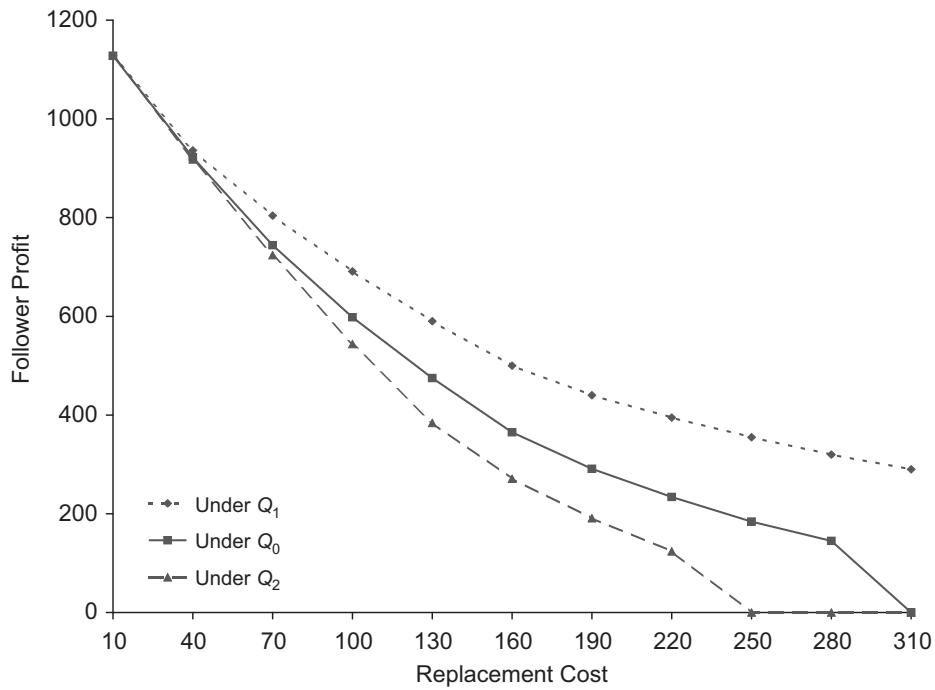


Fig. 3. The follower's expected profits when the leader sells her product at a price of 32.

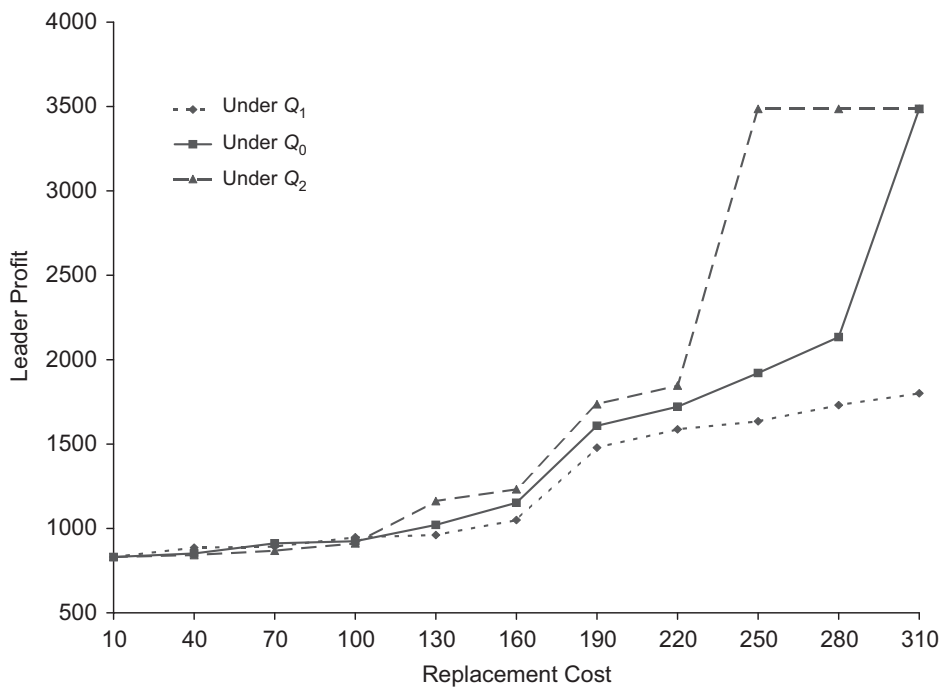


Fig. 4. The leader's expected profits when she sells her product at a price of 32.

acceleration of technology evolution is to lower the product replacement threshold. Table 2, however, illustrates that under each product replacement cost the product replacement threshold does not decrease at all but could actually increase as the pace of technology evolution accelerates. Digging deeper into the data, we find that the observation from Table 2 does not conflict with intuition. We take the case with replacement cost of 280 to illustrate the point further. For a product with zero

performance gap, Table 3 lists, for each technology evolution process, the probability of having the product replaced in the next N stages. Among all three technology processes, the fastest technology process Q_2 had the highest product replacement threshold, but it also has the shortest expected product lifecycle. Table 2 illustrates that the same pattern applies to all other replacement costs. In other words, as the technology evolution accelerates, the technology follower should replace the product more

Table 2
Threshold pattern under different technology evolution.

K_F	Under Q_1		Under Q_0		Under Q_2	
	\bar{v}_4	$E[N]$	\bar{v}_4	$E[N]$	\bar{v}_4	$E[N]$
10	1	1.00	1	1.00	1	1.00
40	2	1.70	2	1.20	2	1.10
70	2	1.70	3	1.54	3	1.21
100	3	2.29	3	1.54	4	1.33
130	3	2.29	4	1.97	5	2.06
160	4	2.87	5	2.42	6	2.23
190	4	2.87	5	2.42	6	2.23
220	5	3.51	6	2.78	7	2.41
250	5	3.51	7	3.18	∞	∞
280	6	4.14	8	3.59	∞	∞
310	6	4.14	∞	∞	∞	∞

Table 3
Probability and expectation of product lifecycles.

	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 6$	$E[N]$
Under Q_1	0.1	0.27	0.287	0.343	0.0	0.0	2.87
Under Q_0	0.0	0.63	0.326	0.0424	1.6×10^{-3}	0.0	2.40
Under Q_2	0.0	0.78	0.21	9.5×10^{-4}	4.9×10^{-5}	1×10^{-5}	2.25

frequently but not by lowering its product replacement threshold blindly.

5. Conclusion

In this paper, we have presented a decision framework for managing a generational product family under technology evolution. The model considers the interaction among technology evolution, product replacement cost and product profitability. Technology evolution follows a continuous Markov process, each product replacement incurs a fixed cost, and a product's profit rate depends on the performance gap between its underlying technology and the latest technology in the market.

As the pace of technology evolution accelerates, many firms face the pressure to raise the frequency of their product replacements and, quite often, see a decline in their profitability. Our decision framework deals with this acceleration pressure from two aspects. First, for a product family with a given product replacement cost, a firm could follow threshold-based replacement policies that guarantee maximum expected profit. Second, when the pace of technology evolution accelerates, instead of simply raising the frequency of its product replacement to follow technological trends, the decision framework shows that the firm should put more emphasis on initiatives that can lower product replacement cost. In fact, in fast-changing industries, firms should strive to cut their product replacement costs through initiatives such as modular and platform-based product design, parallel product development, and flexible manufacturing.

We also construct an analytical model in which innovations from a technology leader drive the technology

evolution process and a technology follower needs to make a product replacement and pricing decisions upon the arrival of each innovation from the technology leader. Consumers' choices between the leader's and the follower's products are given by the MNL model. Numerical examples from the analytical model offers insights on strategies for players in fast-changing industries. In particular, the technology leader can effectively suppress the follower's profit by establishing barriers to raise the follower's product replacement cost. Although speeding up the innovation pace can also help the leader to compete with the follower, such a strategy could result in a lose-lose situation, especially when the follower's product replacement cost is low. For the technology follower, our numerical examples confirm the optimality of threshold-based policies and the benefit from reducing product replacement cost. When the leader speeds up her innovation process, instead of blindly lowering his product replacement threshold to follow the leader's innovation, the follower should keep his emphasis on reducing his product replacement cost.

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Appendix

Proof of Proposition 1. We first prove part (1) of the proposition by contradiction. Suppose that the result is false, then we must have $v_i^* > v_i$, $v_i^* \in P_i$ for some $i \leq (1, n]$ along the sample path. We define \bar{k}_1 as

$$\bar{k}_1 \equiv \begin{cases} n + 1 & \text{if } v_j^* = v_i^* + \sum_{k=i}^{j-1} z_k, \\ & \forall j \in (i, n], \\ \min\{j | v_j^* \neq v_i^* + \sum_{k=i}^{j-1} z_k, j \in (i, n]\} & \text{otherwise.} \end{cases}$$

We construct a decision set \hat{V}_n by setting $\hat{v}_j = v_j^*$ for $j \in [1, (i - 1)] \cup [\bar{k}_1, n]$, $\hat{v}_i = v_i < v_i^*$, and $\hat{v}_j = \hat{v}_i + \sum_{k=i}^{j-1} z_k < v_j^*$ for $j \in (i, \bar{k}_1)$. Because $R(v)$ is decreasing in v , we have $R(\hat{v}_j) > R(v_j^*)$ for $j \in [i, \bar{k}_1)$. That is, following policy \hat{V}_n increases the profit rate for $j \in [i, \bar{k}_1)$.

Now we show that following \hat{V}_n incurs less product replacement cost than following V_n^* . Notice that the difference is from product replacements performed between stage $i - 1$ and stage \bar{k}_1 . First, when $\bar{k}_1 = n + 1$, policy \hat{V}_n does not require any product replacement after stage $i - 1$; policy V_n^* demands at least one product change from v_{i-1}^* to v_i^* . Second, when $\bar{k}_1 < n + 1$, policy \hat{V}_n requires only one product change from $\hat{v}_{\bar{k}_1-1}$ to $v_{\bar{k}_1}^*$; policy V_n^* demands at least two product changes, one from $v_{i-1}^* + z_{i-1}$ to v_i^* and another from v_i^* to $v_{\bar{k}_1}^*$. Combining the above analysis, we can conclude that following \hat{V}_n also reduces

product replacement costs. As a result, \hat{V}_n achieves higher total expected profit than V_n^* does. The contradiction follows the assumption that V_n^* is an optimal product replacement sequence, which concludes the proof for part (1) of the proposition.

We prove part (2) of the proposition also by contradiction. From part (1), we must have $v_i^* \leq v_i$. Suppose that we have $v_i^* \in P_i$ such that $0 < v_i^* < v_i$. Since $v_i^* < v_i$, we have a product replacement at stage i . Let \bar{k}_2 denote the closest stage after stage i that has a product replacement. We can construct another replacement policy \hat{V}_n along the sample path by letting $\hat{v}_j = v_j^*$ for $j \in [1, i - 1] \cup [\bar{k}_2, n]$, $\hat{v}_i = 0$, and $\hat{v}_j = \sum_{k=i}^{j-1} z_k$ for $j \in (i, \bar{k}_2)$. When $j \in [i, \bar{k}_2)$, we have $v_j^* = v_i^* + \sum_{k=i}^{j-1} z_k > \hat{v}_j$, hence $R(v_j^*) \leq R(\hat{v}_j)$. From the assumption that the product replacement cost is fixed at K_F , policy \hat{V}_n incurs the same product replacement cost as does policy V_n^* . Hence, expected profit under \hat{V}_n is higher than that under V_n^* , which contradicts the optimality of V_n^* . \square

Proof of Proposition 2. We prove the results by induction. First, at stage n , we have

$$W_n^k(v_n) = \beta_1 R(v_n) + \beta_2 \sum_{z \in Z} q_z H(v_n + z),$$

$$W_n^r = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z H(z),$$

$$W_n(v_n) = W_n^r - W_n^k(v_n),$$

$$J_n(v_n) = \max\{0, W_n(v_n)\} + W_n^k(v_n).$$

From our assumption, $R(v_n)$ is decreasing in v_n and $H(v_n + z)$ is decreasing in $(v_n + z)$, which leads to $W_n^k(v_n)$ is decreasing in v_n . For part (2), W_n^r is independent of v_n since neither $R(0)$ nor $H(z)$ depends on v_n . Because of $W_n(v_n) = W_n^r - W_n^k(v_n)$, combining part (1) and (2) gives us part (3) for stage n .

Recall from Eq. (6) that $J_n(v_n) = \max\{W_n^k(v_n), W_n^r\}$. Since $W_n^k(v_n)$ is decreasing in v_n and W_n^r is independent of v_n , we have $J_n(v_n)$ is decreasing in v_n , which shows part (4) for stage n .

From the definition of \bar{v}_n given in Eq. (7), we have $W_n(v_n) \geq 0$ if $v_n \geq \bar{v}_n$ and $W_n(v_n) < 0$ otherwise. As a result, $J_n(v_n) = W_n^r$ if $v_n \geq \bar{v}_n$ and $J_n(v_n) = W_n^k(v_n)$ otherwise, which also indicates that $v_n^* = 0$ if $v_n \geq \bar{v}_n$ and $v_n^* = v_n$ otherwise. This implies part (5) for stage n .

For an induction argument we assume that the results are true for stage $i + 1$. At stage i , we have

$$W_i^k(v_i) = \beta_1 R(v_i) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v_i + z),$$

$$W_i^r = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(z),$$

$$W_i(v_i) = W_i^r - W_i^k(v_i),$$

$$J_i(v_i) = \max\{0, W_i(v_i)\} + W_i^k(v_i).$$

Notice from the above equations that for stage i we have $J_{i+1}(v + z)$, $v \in \{0, v_i\}$ instead of $H(v)$. In our proof for stage n , we used the property that $H(v)$ is decreasing in v . From our induction argument, $J_{i+1}(v + z)$ has the above property

due to part (4). Results for stage i follow the same arguments used for stage n , which concludes the induction. \square

Proof of Proposition 3. If $X_{s \geq st} X'_s$, then $P\{X_s \leq i\} \leq P\{X'_s \leq i\}$ for all $i \in I$ and $i > s$. Notice that $P\{X_s \leq i\} = \sum_{z \in Z} \Pr\{s + z \leq i\} = \sum_{z \in Z, z \leq i-s} q_z$. Similarly, $P\{X'_s \leq i\} = \sum_{z \in Z} \Pr\{s + z \leq i\} = \sum_{z \in Z, z \leq i-s} q'_z$. From the definition $Z_{i-s} \equiv \{z | z < i - s, z \in Z \text{ and } i - s \in Z\}$ and $q'_{i-s} = q_{i-s} = 0$ for all $i - s \notin Z$, we have $\sum_{z \in Z, z \leq i-s} q'_z = \sum_{z \in Z_{i-s}} q'_z$ and $\sum_{z \in Z, z \leq i-s} q_z = \sum_{z \in Z_{i-s}} q_z$. Hence, $X_{s \geq st} X'_s$ leads to $\sum_{z \in Z_{i-s}} q_z \leq \sum_{z \in Z_{i-s}} q'_z$. Reversing the order of the above proof, we can show $X_{s \geq st} X'_s$ if $\sum_{z \in Z_{i-s}} q_z \leq \sum_{z \in Z_{i-s}} q'_z$. \square

Proof of Proposition 4. We prove this by induction. First, at stage n , under X we have

$$J_n(v_n) = \max_{v \in \{0, v_n\}} \left\{ -\delta(v_n - v)K_F + \beta_1 R(v) + \beta_2 \sum_{z \in Z} q_z H(v + z) \right\}.$$

Let v_n^* be the optimal choice on performance gap for the above problem, then

$$\begin{aligned} J_n(v_n) &= -\delta(v_n^* - v)K_F + \beta_1 R(v_n^*) + \beta_2 \sum_{z \in Z} q_z H(v_n^* + z) \\ &\leq -\delta(v_n^* - v)K_F + \beta_1 R(v_n^*) + \beta_2 \sum_{z \in Z} q'_z H(v_n^* + z) \\ &\leq J'_n(v_n). \end{aligned}$$

The first inequality is because of $\sum_{z \in Z} q_z H(v_n^* + z) \leq \sum_{z \in Z} q'_z H(v_n^* + z)$, which comes from $X_{\geq st} X'$ and $H(v_n^* + z)$ is decreasing in $v_n^* + z$; the second inequality is because v_n^* is not necessarily an optimal choice under X' .

Now we make an induction argument for stage $i + 1$. Let v_i^* denote the optimal product choice for stage i under technology evolution process X ; then we have

$$\begin{aligned} J_i(v_i) &= \max_{v \in \{0, v_i\}} \left\{ -\delta(v_i - v)K_F + \beta_1 R(v) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v + z) \right\} \\ &= -\delta(v_i^* - v)K_F + \beta_1 R(v_i^*) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(v_i^* + z) \\ &\leq -\delta(v_i^* - v)K_F + \beta_1 R(v_i^*) + \beta_2 \sum_{z \in Z} q'_z J_{i+1}(v_i^* + z) \\ &\leq -\delta(v_i^* - v)K_F + \beta_1 R(v_i^*) + \beta_2 \sum_{z \in Z} q'_z J'_{i+1}(v_i^* + z) \\ &\leq J'_i(v_i). \end{aligned}$$

In the above argument, the first inequality occurs because $X_{\geq st} X'$ and $J_{i+1}(v_i^* + z)$ is decreasing in $(v_i^* + z)$ (part (4) of Proposition 2). The second inequality is from the induction argument, and the third inequality occurs because v_i^* is not necessarily an optimal choice under X' . \square

Proof of Proposition 5. From the threshold definition in Eq. (7), \bar{v}_n and \bar{v}'_n depend on $W_n(v_n)$ and $W'_n(v_n)$ respectively. From the definition, we have

$$\begin{aligned} W_n(v_n) &= -K_F + \beta_1 (R(0) - R(v_n)) \\ &\quad + \beta_2 \sum_{z \in Z} q_z (H(z) - H(v_n + z)), \end{aligned}$$

$$\begin{aligned} W'_n(v_n) &= -K_F + \beta_1 (R(0) - R(v_n)) \\ &\quad + \beta_2 \sum_{z \in Z} q'_z (H(z) - H(v_n + z)). \end{aligned}$$

Notice that the difference between $W_n(v_n)$ and $W'_n(v_n)$ depends on the difference between $\sum_{z \in Z} q_z(H(z) - H(v_n + z))$ and $\sum_{z \in Z} q'_z(H(z) - H(v_n + z))$. When $H(\cdot)$ is convex, $(H(z) - H(v_n + z))$ is decreasing in z . Since $X_{\geq st} X'$, we have $\sum_{z \in Z} q_z(H(z) - H(v_n + z)) \leq \sum_{z \in Z} q'_z(H(z) - H(v_n + z))$, which indicates $W_n(v_n) \leq W'_n(v_n)$, hence $\bar{v}_n \geq \bar{v}'_n$. On the other hand, when $H(\cdot)$ is concave, $(H(z) - H(v_n + z))$ is increasing in z . From $X_{\geq st} X'$, we have $\sum_{z \in Z} q_z(H(z) - H(v_n + z)) \geq \sum_{z \in Z} q'_z(H(z) - H(v_n + z))$, which indicates $W_n(v_n) \geq W'_n(v_n)$, hence $\bar{v}_n \leq \bar{v}'_n$. \square

Proof of Proposition 6. At stage i , we have

$$W_i(v_i) = -K_F + \beta_1(R(0) - R(v_i)) + \beta_2 \sum_{z \in Z} q_z(J_{i+1}(z) - J_{i+1}(v_i + z)),$$

$$W'_i(v'_i) = -K_F + \beta_1(R(0) - R(v'_i)) + \beta_2 \sum_{z \in Z} q'_z(J'_{i+1}(z) - J'_{i+1}(v'_i + z)).$$

From our assumption on $R(\cdot)$, $R(z) \geq R(v)$ for all $v \neq 0$, and $v \in P_i$ hence $\beta_1(R(0) - R(v)) \geq \beta_1(R(0) - R(z)) \geq K_F$. Since $J_{i+1}(z) - J_{i+1}(v_i + z) \geq 0$, we have $W_i(v_i) \geq 0$, hence $J_i(v_i) = W'_i$, indicating that $J_i(v_i)$ is independent of v_i . Applying the same arguments under process X' , we have $W'_i(v'_i) \geq 0$ and $J'_i(v'_i) = W'_i$.

Now we prove $J_i(v_i) = J'_i(v'_i)$ through induction. At stage n , we have

$$J_n(v_n) = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z H(z),$$

$$J'_n(v'_n) = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q'_z H(z).$$

Since $\sum_{z \in Z} q_z H(z) = \sum_{z \in Z} q'_z H(z)$, we have $J_n(v_n) = J'_n(v'_n)$, which concludes our arguments for stage n .

Taking an induction argument for stage $i + 1$, at stage i we have

$$J_i(v_i) = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q_z J_{i+1}(z),$$

$$J'_i(v'_i) = -K_F + \beta_1 R(0) + \beta_2 \sum_{z \in Z} q'_z J'_{i+1}(z).$$

The result for stage i comes directly from that both $J_{i+1}(z)$ and $J'_{i+1}(z)$ are independent of z and $J_{i+1}(z) = J'_{i+1}(z)$, which concludes the proof. \square

Proof of Proposition 7. To prove part (1), we show that $\partial R(v, p_i) / \partial p_i$ crosses zero exactly once. Taking partial derivative of $R(v, p_i)$, we have

$$\frac{\partial R(v, p_i)}{\partial p_i} = N(v, p_i) D_i + (p_i - C(v)) \frac{\partial N(v, p_i)}{\partial p_i} D_i = N(v, p_i) D_i [1 - (p_i - C(v)) w_p (1 - N(v, p_i))].$$

Since $p_i^*(v)$ satisfies $\partial R(v, p_i) / \partial p_i = 0$ and $N(v, p_i) > 0$, we must have $(p_i^*(v) - C(v)) w_p (1 - N(v, p_i^*(v))) = 1$. Let $G(p_i) \equiv (p_i - C(v)) w_p (1 - N(v, p_i))$. Taking $G(p_i)$'s partial derivative on p_i , we have

$$\frac{\partial G(p_i)}{\partial p_i} = w_p (1 - N(v, p_i)) (1 + (p_i - C(v)) N(v, p_i)) > 0.$$

Therefore, $G(p_i) < 1$ when $p_i \in (C(v), p_i^*(v))$, and $G(p_i) > 1$ when $p_i \in (p_i^*(v), \infty)$. As a result, $\partial R(v, p_i) / \partial p_i > 0$ when $p_i \in (C(v), p_i^*(v))$, and $\partial R(v, p_i) / \partial p_i < 0$ when $p_i \in (p_i^*(v), \infty)$. This shows $\partial R(v, p_i) / \partial p_i$ crosses zero at and only at $p_i^*(v)$, hence $p_i^*(v)$ maximizes $R(v, p_i)$.

To prove part (2), we examine the condition that $p_i^*(v)$ satisfies $(p_i^*(v) - C(v)) w_p (1 - N(v, p_i^*(v))) = 1$ for all $v > 0$. Suppose that we have $v^1 > v^2$ but $p_i^*(v^1) > p_i^*(v^2)$. Since $C(v)$ is decreasing in v , we must have $p_i^*(v^1) - C(v^1) > p_i^*(v^2) - C(v^2)$. Notice that $N(v, p_i)$ is decreasing in v and p_i , and we have $N(v^1, p_i^*(v^1)) < N(v^2, p_i^*(v^2))$. As a result, we have

$$(p_i^*(v^1) - C(v^1)) w_p (1 - N(v^1, p_i^*(v^1))) > (p_i^*(v^2) - C(v^2)) w_p (1 - N(v^2, p_i^*(v^2))),$$

which contradicts the condition $(p_i^*(v) - C(v)) w_p (1 - N(v, p_i^*(v))) = 1$. Therefore, we must have $p_i^*(v)$ is decreasing in v .

To prove part (3), we evaluate the derivative of $N(v, p_i^*(v))$ on v , which is given by

$$\frac{dN(v, p_i^*(v))}{dv} = \frac{\partial N(v, p_i^*(v))}{\partial v} + \frac{\partial N(v, p_i^*(v))}{\partial p_i^*(v)} \frac{dp_i^*(v)}{dv} = -N(v, p_i^*(v)) (1 - N(v, p_i^*(v))) \left(w_t - w_p \frac{dp_i^*(v)}{dv} \right).$$

From the proof for part (1), we know that $p_i^*(v)$ must satisfy $(p_i^*(v) - C(v)) w_p (1 - N(v, p_i^*(v))) = 1$. Taking the derivative with respect to v on both sides and resequencing the terms, we have

$$\frac{dp_i^*(v)}{dv} = - \frac{(p_i^*(v) - C(v))}{(1 - N(v, p_i^*(v)))} \frac{dN(v, p_i^*(v))}{dv} - \gamma.$$

Bringing the above term into $dN(v, p_i^*(v)) / dv$, we get

$$\frac{dN(v, p_i^*(v))}{dv} = - \frac{w_p N(v, p_i^*(v)) (1 - N(v, p_i^*(v)))}{1 + w_p (p_i^*(v) - C(v)) N(v, p_i^*(v))} \left(\frac{w_t}{w_p} - \gamma \right).$$

Therefore, $dN(v, p_i^*(v)) / dv \leq 0$ when $\gamma \leq w_t / w_p$ and $dN(v, p_i^*(v)) / dv > 0$ otherwise, which proves part (3).

To prove part (4), we take the partial derivative of $R(v, p_i)$ on v .

$$\frac{\partial R(v, p_i)}{\partial v} = \frac{\partial C(v)}{\partial v} N(v, p_i) D_i - (p_i - C(v)) \frac{\partial N(v, p_i)}{\partial v} D_i = \gamma N(v, p_i) D_i - (p_i - C(v)) [w_t N(v, p_i) (1 - N(v, p_i))] D_i.$$

Applying the Envelope theorem, we evaluate $\partial R(v, p_i) / \partial v$ at $p_i^*(v)$, which gives us

$$\begin{aligned} \frac{\partial R(v, p_i)}{\partial v} \Big|_{p_i^*(v)} &= \gamma N(v, p_i^*(v)) D_i \\ &\quad - p_i^*(v) [w_t N(v, p_i^*(v)) (1 - N(v, p_i^*(v)))] D_i \\ &\quad + C(v) [w_t N(v, p_i^*(v)) (1 - N(v, p_i^*(v)))] D_i \\ &= \gamma N(v, p_i^*(v)) D_i - \frac{w_t}{w_p} N(v, p_i^*(v)) D_i, \end{aligned}$$

where the second equality is from $(p_i^*(v) - C(v)) w_p (1 - N(v, p_i^*(v))) = 1$. The result follows $\partial R(v, p_i) / \partial v|_{p_i^*(v)} \leq 0$ when $\gamma \leq w_t / w_p$ and $\partial R(v, p_i) / \partial v|_{p_i^*(v)} > 0$ otherwise. \square

Proof of Proposition 8. Notice that product replacement cost is K_F and $R(\nu)$ is decreasing in ν when $\gamma \leq w_t/w_p$. The result follows since all assumptions for Proposition 2 are satisfied. \square

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