

## Chapter 5



# OPTIMAL USE OF DEMAND INFORMATION IN SUPPLY CHAIN MANAGEMENT

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### 1. Introduction

Alan Greenspan was puzzled by the data on his computer screen. Capital expenditures in high technology were rising sharply, unemployment was declining, prices were holding steady and profits were rising. At the same time, the Labor Department's statistics showed that productivity had decreased by one percent during the second quarter. Greenspan did not agree with the productivity data. He found the missing link in the Survey of Current Business from the Bureau of Labor Statistics, Woodward [44], which indicated that business inventories were shrinking significantly while the economy was growing. This suggested that new computer technology was allowing just-in-time orders. Instead of stocking products weeks or months in advance, businesses could keep detailed track of what was needed and order within days, while competitive pressures were forcing quality control. Greenspan eventually became convinced by this argument and voiced it publicly in the *State of the Economy* address before the Committee on Ways and Means, U.S. House of Representatives, January 20, 1999.

Are companies really using demand information optimally to manage inventories? If so, what models are driving these decisions? Anecdotal evidence suggest that the implementation of Enterprise Resource Planning (ERP) software, such as SAP, Baan has allowed companies to integrate data bases and greatly improve the quality of the data that resides therein. In addition, the purchase and implementation of numerous ERP bolt-ons, such as i2 and Manugistics, were justified, at least in part, in integrating manufacturing and replenishment decisions with key suppliers and basing these decisions on customers' needs.

Capital investments in information technology have certainly provided members of supply chain networks with more timely and better quality data than ever before. As members of supply networks, inventory managers need to decide what data to collect, request, or buy from their customers, and what data to pass on, or sell to their suppliers. In addition, inventory managers need to develop effective replenishment policies that make full use of demand information.

This chapter is concerned with models where current demand information is used to drive inventory replenishment policies in distributed decision making settings and with advance demand information models where customers place orders in anticipation of future requirements. The problem of using current demand information on single echelon systems has been studied extensively and reviewed by Scarf [38], Veinott [43], and Porteus [35]. Recent papers in this area are concerned with the use of current demand information in a supply chain context. A typical example is a retailer having point of sale (POS) information that is valuable to his supplier. Questions that arise include: What is the optimal policy for the supplier when he only sees orders from the retailer? What is the optimal policy for the supplier when in addition to retail orders he has access to POS information? What is the value of POS information to the supplier? Is the retailer better off by sharing POS information? If not, can POS information be traded so that both the retailer and the supplier are better off? Authors that have studied different versions of this problem include Bourland et al. [4], Chen [6], Gallego et al. [16], Gavirneni et al. [21], and Lee et al. [30].

Models of advance demand information assume that the total demand for a period can be expressed as the sum of orders placed over a certain horizon prior to the period. For example, the total demand for period five may be the sum of orders placed in periods two, three, four, and five. There are two motivations for studying demand models of this form. First, under long term contracts, downstream supply chain partners often agree to forecast and update future requirements and to freeze orders within a certain time window. Ford Motor Company, for

example, issues orders and updates these orders for the next few weeks to its catalytic converter supplier as discussed in the Harvard Business School teaching case “Corning Glass Works” (1991). Long term contracts, however, are not the only way companies get advance demand information. Advance demand information also occurs when customers place orders in advance, perhaps through the Internet, for customized products such as a upscale furniture and computer equipment.

The second reason to study models with advance demand information is that there is a growing consensus, see Appell et al. [1], that manufacturers can benefit from having a portfolio of customers with different demand leadtimes<sup>1</sup>. A portfolio of customers with different demand leadtimes can enable better capacity utilization, higher and more consistent revenues and better customer service.

Customers often need to be induced to place advance orders. Chen [7] provides an example where market segmentation is used to gain advance demand information. The problem of finding price structures to induce advance bookings has a revenue management flavor, Gallego and van Ryzin [20], and is outside the scope of this chapter. Advance demand information inventory models, however, are very complementary to the problem of finding effective incentives for customers to book in advance. Indeed, the advance demand information models studied in this chapter are about finding effective inventory policies and about evaluating the cost benefits of advance demand information. Without the ability to assess the cost benefits of advance demand information it is very difficult to design cost effective incentives to induce advance bookings. Authors who have studied models related to the concept of advance demand information include Gallego and Özer [17], [18], Gallego and Toktay [19], Graves et al. [22], Güllü [23], [24], Hariharan and Zipkin [26], Heat and Jackson [28], Toktay and Wein [42], Özer [33] Schwarz, et al. [40], and Sethi et al. [41].

The aim of this chapter is to summarize the result in Gallego, et al. [16], Gallego and Özer [17], [18], and Özer [33]. In the course of summarizing these papers we also review the work of other authors that has contributed to our understanding of these problems. Section 2 deals with models using current demand information, while Section 3 deals with models using advance demand information.

## 2. Using Current Demand Information

This section will deal mostly with multi-location models using current demand information. However, we start our discussion with a single location model due to Herbert Scarf. His model is an example

where demand information is used to decide whether or not to exercise an option. After presenting Scarf's model we will discuss papers by Bourland et al. [4], Chen [6], Gavirneni et al. [21], Gallego et al. [16], and Lee et al. [30]. These authors find that sharing *current* demand information in supply chains is valuable when order cycles are asynchronous, when information is centralized in a vertically integrated system, when capacity is limited, when the retailer orders in batches to take advantage of economies of scales, and when the demand process is autocorrelated.

### 2.1. Optimal inventory policies when sales are discretionary

Herbert Scarf is well known for his early contributions to inventory theory, see Clark and Scarf [11], Scarf [37], and [38]. After a 40 years hiatus, Scarf [39] returns to the field to investigate the nature of optimal policies when sales are discretionary, i.e., when the inventory manager can choose to meet a fraction of the demand that arises during any given period. Inventory managers may exercise the discretionary sales option in anticipation of higher prices or higher costs.

In Scarf's model the horizon is assumed to be finite, ordering costs are fixed plus linear, lead times are zero, and unsatisfied demands are lost. The demand during the period is a random variable  $D$ . The special feature of this model is that the manager may choose to sell any quantity  $z$  with  $0 \leq z \leq \min[y, D]$  at the market price  $p$  where  $y$  is the period's starting inventory after ordering. The ordering cost, selling price and demand distributions can be time varying aside from the requirement that the set-up costs decrease monotonically over time. The objective is to maximize expected discounted profit.

Scarf's analysis makes use of a novel property of  $K$ -concave functions<sup>2</sup>. Scarf shows that if  $d \geq 0$  is a constant,  $f$  is a  $K$ -concave function defined on  $[0, \infty)$  with a finite number of local maxima,

$$g(y) = \max_{0 \leq z \leq \min[y, d]} f(y - z),$$

and  $z(y)$  is the largest minimizer of  $f(y - z)$  subject to  $0 \leq z \leq \min[y, d]$ , then  $g$  is  $K$ -concave and  $y - z(y)$  is monotonically increasing in  $y$ .

Scarf uses this property to show that policy that maximizes the expected discounted profit is of the  $(s, S)$  form. Under this policy the inventory manager observes the inventory  $x$  and places an order for  $S - x$  units if  $x \leq s$  and does not order otherwise. After observing the demand, say  $d$ , during the period, the inventory manager sells  $z(y) \in [0, \min(y, d)]$

and keeps  $y - z(y)$  units for the next period. While speculating with inventories is rarely seen in stationary environments, this behavior is typical in inflationary environments and in environments where there is significant price and or cost volatility.

## 2.2. The value of demand information when order cycles are asynchronous

Bourland et al. [4] consider a two stage serial system where each stage follows a periodic review base-stock policy. Each stage is assumed to have ordering periods of equal length. The authors investigate the benefits of sharing customer demand information through EDI when the ordering cycles are *not* synchronized. As an example, consider a supplier and a retailer making ordering decisions on a weekly basis. Assume the retailer places orders every Thursday and the supplier places orders every Monday. Assume that the supplier incurs linear holding and back-order costs. In the absence of demand information the retailer faces a periodic review inventory problem with iid demands. Under this condition the supplier's expected cost is proportional to standard deviation of the weekly demand. Suppose, instead, that on Mondays the supplier obtains information from the sales experienced by the retailer since Thursday. Thus, at the time of placing the order, the supplier only faces uncertainty about the demand for Monday, Tuesday, and Wednesday. The cost to the supplier is now proportional to the standard deviation of the demand over these three days. If the demand is uniform over the week then the expected cost under demand information sharing is 65.4% ( $\sqrt{3/7}$ ) of the expected cost in the absence of demand information sharing.

## 2.3. The value of demand information when capacity is limited

Gavirneni et al. [21] study the holding and penalty cost of a finite capacity supplier facing demands from a single retailer following an  $(s, S)$  policy. The authors assume that the retailer's lead time is zero and that the retailer can immediately procure units from an alternative source if the supplier is not capable of filling an order. The supplier absorbs the assumed linear cost of procuring such units. Under these assumptions, the authors compare the costs with and without information sharing, and report significant savings for the supplier from demand information sharing. The savings come from the supplier's increased ability

to anticipate the timing and the magnitude of the next order from the retailer.

#### 2.4. The value of demand information when the retailer batches orders

Gallego et al. [16] assess the benefits of sharing demand information in a supply chain consisting of a single supplier and a single retailer. The retailer faces Poisson demands at rate  $\lambda$ , economies of scale in ordering, and places orders from the supplier according to a  $(Q, r)$  policy where  $Q$  is fixed.<sup>3</sup> The supplier orders from an outside source with ample stock and incurs linear holding and backlogging costs at rates  $h$  and  $p$  respectively. Units ordered from the outside source arrive at the supplier after  $L$  units of time, where  $L$  is a known constant. The supplier's problem is to minimize the expected holding and backorder penalty costs with or without retail demand information. The authors show that the supplier's optimal policy under demand information sharing calls for monitoring the retailer's inventory position and results in a cost that is independent of the order size  $Q$ . In the absence of demand information they show that a modified base-stock policy where the supplier introduces a random delay after receiving the order from the retailer is optimal. The authors then investigate whether or not the retailer is better off by voluntarily sharing demand information. When this is not the case, the authors identify conditions under which information can be traded, i.e., purchased by the supplier.

In the absence of demand information the supplier only observes orders of size  $Q$  from the retailer. The time between orders observed by the supplier is an Erlang random variable, say  $T_Q$ , with parameters  $Q$  and  $\lambda$ . The authors assume that the supplier knows the distributional form of  $T_Q$  or equivalently that the supplier knows, or can accurately estimate, the demand rate  $\lambda$ . The expected cost of the supplier's optimal base-stock policy is given by  $\min H(mQ)$  where the minimization is done over integer values of  $m$ ,  $H(y) = hE(y - N_L)^+ + pE(N_L - y)^+$  and  $N_t$  denotes a Poisson random variable with parameter  $\lambda t$ . They show that the largest optimal base stock level is  $S = m_o Q$  where  $m_o$  is given by

$$m_o = \min \left\{ m : m \in \mathcal{Z}^+, \frac{1}{Q} \sum_{k=mQ+1}^{(m+1)Q} P(N_L < k) > \frac{p}{h+p} \right\}. \quad (5.1)$$

The case  $Q = 1$  is of special interest, since in this case the retail orders coincide with the demand seen by the retailer. Thus, in this case, the supplier has full demand information. The optimal base stock level, say

$S^*$ , is given by the smallest integer  $S$  satisfying

$$P(N_L \leq S) > \frac{p}{h+p}. \quad (5.2)$$

Obviously,  $H(S^*) \leq H(m_o Q)$  for all  $Q > 1$ . In fact  $H(S^*)$  is a lower bound on the optimal with respect to all policies for all  $Q$ , even if the supplier has full demand information.

#### 2.4.1 Optimal policies under demand information sharing.

When the retailer shares demand information with the supplier, the supplier's optimal policy consists of monitoring the retailer's inventory position and placing an order of size  $Q$  when the retailer's inventory position drops to  $r+n$  where  $n = S^* - (m-1)Q$  and  $m$  is an integer such that  $(m-1)Q < S^* \leq mQ$ . The expected cost to the supplier under this policy is shown to be equal to  $H(S^*)$ , and is independent of  $Q$ . A numerical comparison shows that the cost of this policy can be significantly lower than the cost of the best base-stock policy. For example, for  $\lambda = 20, L = 1, h = 1$ , and  $p = 9, S^* = 26$ , and  $H(26) = 8.19$ . On the other hand, for  $Q = 15, m_o = 2$  and  $H(30) = 10.32$  while for  $Q = 20, m_o = 1$  and  $H(20) = 17.77$ .

#### 2.4.2 Optimal policies without demand information sharing.

The optimal policy under demand information sharing *delays* the placement of orders until  $Q - n$  units are demanded. This suggests that expanding the class of base-stock policies by allowing the supplier to delay orders. We first consider base-stock policies with fixed and then random delays. Gallego et al. show that random delay base-stock policies are optimal.

If the supplier delays orders arriving from the retailer by  $\tau$  units of time then the system behaves as if the lead time was  $L + \tau$  instead of  $L$ . We can write the cost of delayed base-stock policies as  $H(mQ, \tau) = hE[y - N_{L+\tau}]^+ + pE[N_{L+\tau} - y]^+$ . Fixed delay base-stock policies were recently proposed by Moinzadeh [31]. He computes the optimal delay  $\tau$  for a fixed order size  $mQ$  for the case where the inter-order distribution is normal and for the case where only the mean and variance of the interorder distribution is known. Let

$$m_1 = \min \left\{ m \in \mathbb{Z}^+ : P(N_L < mQ) = P(t_{mQ} > L) > \frac{p}{h+p} \right\}. \quad (5.3)$$

Corresponding to every integer  $m \geq m_1$  there is a unique  $\tau_m > 0$  such that  $(mQ, \tau_m)$  is a stationary point. The existence of a countable

number of stationary points lead Moinzadeh to an extensive computational search for an optimal solution. Gallego et al. show that it is enough to consider only the points  $(m_o, 0)$  and  $(m_1, \tau_1)$  where  $\tau_1$  is the unique positive root of  $P(t_{mQ} > L + \tau) = P(N_{L+\tau} < mQ) = \frac{p}{h+p}$ .

**2.4.3 Random delay base-stock policies.** The authors consider a policy that terminates the delay if the next order from the retailer arrives before the fixed delay. Let  $\nu$  be the fixed delay and  $T_Q$  the time until the next order from the retailer. Under the random delay base-stock policy, the supplier delays her order by  $\min(\nu, T_Q)$ . By the reward renewal theorem the average cost of this policy can be written as

$$\begin{aligned} G(mQ, \nu) &= h\lambda E(T_{(m-1)Q} - L + (T_Q - \tau)^+)^+ + p\lambda E(L - T_{(m-1)Q} - (T_Q - \tau)^+)^+ \\ &= hE[(m-1)Q - N_L + (Q - N_\nu)^+]^+ + pE[N_L - (m-1)Q - (Q - N_\nu)^+]^+ \end{aligned}$$

Notice that if  $\nu = 0$  then

$$G(mQ, 0) = H(mQ, 0) = hE[mQ - N_L]^+ + pE[N_L - mQ]^+.$$

Thus, the largest optimal  $m$  for  $\nu = 0$  is  $m_o$  as defined by (5.1). We need to compare the cost  $G(m_oQ, 0) = H(m_oQ, 0)$  to the cost of the best policy with a positive delay. Recall the definition of  $m_1$  in equation (5.3). Let  $\nu_1$  be the unique solution to

$$P(N_L + N_\nu < m_1Q \mid N_\nu < Q) = \frac{p}{h+p}. \quad (5.4)$$

The authors show that the pair  $(m_1Q, \nu_1)$  is the only strictly interior stationary point of  $G(mQ, \nu)$ . Thus, to find the optimal random delay base-stock policy we only need to compare  $G(m_oQ, 0)$  and  $G(m_1Q, \nu_1)$ . Let  $(m^*Q, \nu^*)$  denote the pair with lower cost among these two candidate solutions. The authors also show that policy  $(m^*Q, \nu^*)$  minimizes the supplier's long run average holding and penalty cost among all possible policies by using arguments due to Katircioglu [25].

Although the random delay base-stock policy has the virtue of being provably optimal, our computations indicate that the savings relative to the fixed delay base-stock policy are almost always negligible. The only case where there was a small, but a significant, difference in cost was for  $Q = 20$  where  $\nu_1 = 0.6279$ . This case resulted in an average cost of \$10.19, which is a slight improvement over the cost \$10.33 of using the fixed delay  $\tau_1 = 0.6069$ . As a final observation, note that random delay policies are unlikely to be significantly better than fixed delay policies for small and large values of  $Q$ . Indeed, for small values of  $Q$  we almost

have retail demand information, whereas for large  $Q$  values  $T_Q$  becomes large and almost deterministic.

$Q$	$\nu^*$	$m^*Q$	$G(m_oQ, 0)$	$G(m^*Q, \nu^*)$	$\frac{G(m_oQ, 0) - G((m^*Q, \nu^*))}{G(m_oQ, 0)}$
1	0	26	8.19	8.19	-
5	0	25	8.31	8.31	-
10	0.1615	30	10.32	8.83	14.44%
15	0.1615	30	10.32	8.83	14.44%
20	0.6279	40	17.77	10.19	42.66%
25	0	25	8.31	8.31	-
30	0.1615	30	10.32	8.83	14.44%
40	0.6069	40	20.00	10.33	48.35%
50	1.059	50	30.00	11.64	61.21%

**2.4.4 To share or not to share demand information.**

Suppose it takes  $l$  units of time for a shipment to get from the supplier to the retailer. If we take the view that stockouts at the supplier cause shipment delays to the retailer, then the retailer's lead time demand under demand information sharing is

$$N_l + (N_L - S^*)^+,$$

where  $S^*$  is given by (5.2). On the other hand, the retailer's lead time demand without information sharing is

$$N_l + (N_L - (m - 1)Q - (Q - N_{\nu^*})^+)^+,$$

where  $(m^*Q, \nu^*)$  minimizes  $G$ .

The retailer needs to compute his cost under each of these lead time demands to decide whether or not it is in his best interest to share demand information. For example, when  $Q = 20$ , the retailer would be willing to share demand information. This is a win-win situation since both the retailer and the wholesaler benefit from sharing demand information. On the other hand, for  $Q = 10$  the retailer will be unwilling to share demand information. A similar situation arises when  $Q = 15$  and when  $Q = 30$ . If the retailer is worse off sharing demand information, the supplier may be willing to buy this information from the retailer if the expected gain to the supplier exceeds the expected cost to the retailer. Although we are not aware of any practical instance where the retailer sells demand information for a fee, there may be an implicit cost paid by the supplier in the form of lower unit prices. Moreover, as retailers become more powerful, they may soon be in a position of actually demanding payment for demand information. Finally, if neither selling nor sharing demand is jointly profitable, the supplier and the retailer may

attempt to jointly optimize the sum of their costs and then find a way to share the savings. We note, however, that moving from distributed to centralized decision making is nontrivial since it entails sharing cost structures which may be misrepresented.

### 2.5. The value of centralized demand information in vertically integrated firms

Chen [6] considers a continuous-time model of a vertically integrated serial supply chain where the demand process at the downstream stage is compound Poisson. Each stage is assumed to follow a  $(nQ, r)$  replenishment policy with given batch sizes. Reorder points are computed based on echelon (centralized) and on local (decentralized) information with the objective of minimizing system wide expected costs. It is known that the cost difference is zero when demand is Poisson and the batch sizes are all one, but Chen's is the first comprehensive numerical study to investigate the cost difference under compound Poisson and arbitrary, but fixed, batch sizes. Surprisingly, the cost difference is fairly small, 1.75% on the average, and a maximum of 9%. The cost seems to be larger the longer the lead time, the larger the batch sizes, and the greater the number of echelons in the chain. We remark that the policies found by Chen are optimal among the class of  $(nQ, r)$  policies he considers. It is quite possible that the idea of delaying orders, described in the previous section, may provide a strict improvement over the class of  $(nQ, r)$  policies, but more research is needed to assess the extent to which delay policies are beneficial in this setting.

### 2.6. The value of demand information when demand is autocorrelated

Lee et al. [30] study the benefit of demand information sharing to the manufacturer in a two stage supply chain consisting of a single manufacturer and a single retailer facing an autocorrelated demand process. The manufacturer and the retailer incur linear holding and backlogging costs, experience constant lead times, and follow base-stock policies. The authors assume that the manufacturer immediately ships retail orders, instantly procuring from an alternative source at a linear penalty cost in the case of a shortfall. The authors report large savings for the manufacturer when the autocorrelation coefficient  $\rho \in (-1, 1)$  is high. These savings are computed under the assumption that the manufacturer knows the parameters of the demand process, observes only retail orders, and makes forecasts based only on the *last* retail order.

The demand process is assumed to be given by

$$D_t = d + \rho D_{t-1} + \epsilon_t,$$

where  $d > 0$ ,  $-1 < \rho < 1$ , and  $\epsilon_t$  is iid normally distributed with mean zero and variance  $\sigma^2$ .

Let  $S_t$  denote the order-up-to level in period  $t$ . At the end of time  $t$  the retailer orders

$$Y_t = D_t + (S_t - S_{t-1})$$

which represents the demand during period  $t$  plus the change made in the order-up-to level. The authors acknowledge that it is possible to have  $Y_t < 0$  but they assume  $\sigma \ll d$  so that  $P(Y_t < 0)$  is negligible. The authors show that the conditional expectation and the conditional variance of the total lead time demand are given by

$$m_t = a + bD_t,$$

and

$$v_t = c^2\sigma^2,$$

where  $a$  is a constant that depends on  $d$ ,  $\rho$  and  $l$ , while  $b$  and  $c$  are constants that depend only on  $\rho$  and the length of the lead time  $l$ . Consequently,

$$S_t = m_t + kc\sigma,$$

and

$$Y_t = D_t + b(D_{t+1} - D_t),$$

where  $k$  is the  $p/(h+p)$  percentile of the standard normal distribution.

The authors then solve the manufacturers problem under both information sharing and no information sharing. The authors assume that, in the case of no information sharing, the manufacturer estimates the conditional mean and variance of his lead time demand based on  $Y_t$  only, as opposed to  $D_t$  in the case of demand information sharing. It is assumed that the manufacturer knows  $d$ ,  $\rho$ , and  $\sigma$ . The conditional mean and variance of the manufacturer's lead time demand is then used to set the manufacturer's order-up-to levels. The authors attribute very large savings to the manufacturer's long run average cost, especially for  $\rho > 0.5$ , to demand information sharing.

We find, however, that the manufacturer can estimate  $D_t$  as follows: Let  $\hat{D}_0 = \frac{d}{1-\rho}$ , and for  $t > 0$  let

$$\hat{D}_t = \frac{Y_t + b\hat{D}_{t-1}}{1+b}.$$

Let  $e_t = D_t - \hat{D}_t$  be the estimation error in period  $t$ . A little algebra reveals that

$$\begin{aligned} D_t - \hat{D}_t &= \frac{b}{1+b}(D_{t-1} - \hat{D}_{t-1}) \\ &= \frac{b}{1+b}e_{t-1} \\ &= A^{t-1}e_1, \end{aligned}$$

where  $A = \frac{b}{1+b} = \frac{\rho(1-\rho^{l+1})}{1-\rho^{l+2}} \in (-1, 1)$ , which implies that  $e_t \rightarrow 0$  as  $t \rightarrow \infty$ . Since  $\hat{D}_0$  is the unconditional mean of  $D_t$ , it follows that  $e_1 = \epsilon_1$  is normal with mean zero and variance  $\sigma^2$ . Thus the absolute value of the initial error is, with very high probability, bounded by  $3\sigma \ll d$ . As an example, assume that  $\rho = 0.6$ ,  $d = 100$ ,  $l = 5$ , and  $\sigma = 10$ . Then a large initial error of  $3\sigma = 30$  is 15% of the mean demand. After 5 weeks the error is 1.8% of the mean demand, and after 12 weeks the error is negligible. Armed with the estimate  $\hat{D}_t$  the manufacturer can do as well as under demand information sharing after 12 periods. This analysis shows that the savings in average costs reported by the authors are only transient and are zero in the long run. Demand information sharing is valuable when the manufacturer cannot infer current demand from the order history. This would be the case, for example, if demand were driven, in part, by pricing and promotion activities.

### 3. Using Advance Demand Information

Hariharan and Zipkin [26] incorporate advance demand information in a single echelon continuous review system by introducing the concept of *demand leadtimes*. In their model customers place orders  $l$  periods in advance of their requirements. As a consequence, the inventory manager has perfect information about future demand. For fixed demand lead time,  $L_D$ , they prove the optimality of base stock policies and show that demand lead times directly offset supply leadtimes. As an example, consider the case where the supply lead time is  $L$ . If  $0 \leq L_D \leq L$  the system with demand leadtimes behaves as a system with supply lead time  $L - L_D$ . As  $L_D$  approaches  $L$  the system moves from make-to-stock to make-to-order. If  $L_D > L$ , then the system is make-to-order and it is optimal for the inventory manager to delay orders by  $L_D - L$  units of time.

There have been a number of attempts to generalize the results of Hariharan and Zipkin to the case where advance demand information is not perfect. Schwarz et al. [40] consider a periodic review model where a

retailer has *imperfect* demand information over the demand leadtime  $L_D$ . To better understand their model, consider the special case  $(L_D, L) = (1, 2)$ . At the beginning of a period, the inventory manager observes the potential demand  $Z = z$  for the next period. The demand, say  $X$ , that materializes in the following period depends on  $z$ . For example,  $Z$  may be Poisson with parameter  $\lambda/(1-p)$  and  $X$  binomial with parameters  $Z = z$  and  $1-p \in [0, 1]$  where  $p$  is the cancellation rate. Schwarz et al. consider the problem of maximizing the total expected discounted profit over an infinite horizon for a problem with stationary cost parameters and a stationary demand process under the assumption that inventory manager can order or dispose units at a linear cost. They show that a state dependent base stock policy is optimal where the state is the vector of demand signals over the demand leadtime. The authors compare systems with imperfect information,  $0 < p < 1$  to the extreme cases of perfect and no information, and provide insights about the nature of the optimal policies and the corresponding expected discounted profits. Notice that the sequence of demand forecasts namely,  $\lambda$ ,  $Z(1-p)$  and  $X$  forms a martingale since  $E[Z(1-p)] = \lambda$  and  $E[X|Z(1-p)] = Z(1-p)$  with  $E[X] = E[E[X|Z]] = E[Z(1-p)] = \lambda$ . Viewed this way, the work of Schwarz et al. is best examined in the light of earlier results dealing with demand forecast revisions.

Work on demand forecast revisions include Hausman [27], Heat and Jackson [28], Graves et al. [22], and Güllü [23], [24], Gallego and Toktay [19], and Toktay and Wein [42]. Hausman [27] models the evolution of forecast as a quasi-Markovian or Markovian process. He suggests that ratios of successive forecasts can be modeled as independent lognormal variates and incorporated into sequential decision problems. Heat and Jackson [28] model the evolution of forecast using martingales. They name their model the Martingale Method of Forecast Evolution (MMFE) and use it to generate forecasts for a simulation model and to analyze economic safety stock levels for a multi-product multi-facility production system. Graves et al. [22] independently model a single item version of MMFE and use it to analyze a two stage production planning system. Güllü [23] determines the form of the optimal policy that arises under MMFE for a capacitated single item/single facility inventory system with zero set up cost and instantaneous delivery. He shows that the system, which employs demand forecast one period in the future, attains lower expected minimum cost than a system that does not incorporate future demand information. This result is a precursor to Schwarz et al. in the context of cost minimization that does not make the disposal assumption. Güllü [24] studies the behavior of the optimal order up to policy with respect to capacity levels and the forecast state. Gallego

and Toktay [19] study a production problem with demand updates where capacity is limited and the fixed cost is large enough to justify an all or nothing production policy. Under such a policy, production in a given period is either zero or the maximum capacity. They show that a state dependent threshold policy is optimal. Toktay and Wein [42] model a setting where the production stage is modeled as a single-server discrete-time continuous-state queue. They use heavy traffic and random walk theory to obtain a closed form approximation for the forecast adapted base stock policies where forecasts are updated as in MMFE. Sethi et al. [41] study a model of forecast evolution with zero setup costs and multiple delivery modes and show that a state dependent policy is optimal.

The work that we review in this section, Gallego and Özer [17], Gallego and Özer [18], and Özer [33], deal respectively, with single location, serial, and distribution systems under a model of advance demand information. The advance demand information model is a discrete time generalization of Hariharan and Zipkin, and fits into the MMFE framework in a very explicit way. The idea is that total demand for any given period is the sum of customer commitments made over a certain horizon. For example, the total demand for period five is the sum of commitments made by customers in periods three, four, and five. This generalizes the idea of Hariharan and Zipkin in that part of the demand is known in advance. Our model, unlike that of Schwarz, Petruzzi and Wee, does not allow for cancelations. While this is a limiting factor, as stated in the introduction, there are two reasons to study these models. First, some manufacturers have long term price agreements with customers that place firm orders in advance. Second, there is a growing consensus that manufacturers can benefit from having a portfolio of customers with different demand lead times. Such portfolios lead naturally to inventory systems with advance demand information.

### 3.1. Advance demand information model

In this section we present the advance demand information model in detail. To model advance demand information, we assume that customers place orders during a period. Such orders may be either for immediate delivery or to be delivered at a specified period in the near future. To be more precise, we assume that during period  $t$  we observe the demand vector

$$D_t = (D_{t,t}, \dots, D_{t,t+N}),$$

where  $D_{t,s}$  represents orders placed by customers during period  $t$  for periods  $s \in \{t, \dots, t+N\}$ , where  $N$  represents the length of the *information horizon*.  $N = 0$  represents the classical case of no advance demand information.

At the beginning of period  $t$ , the demand to prevail in period  $s \geq t$  can be divided into two parts: The *observed* part that is known to us

$$O_{t,s} \equiv \sum_{r=s-N}^{t-1} D_{r,s}, \quad (5.5)$$

and the part that is *unobserved* and not yet known to us

$$U_{t,s} \equiv \sum_{r=t}^s D_{r,s}. \quad (5.6)$$

We define  $O_{t,s} \equiv 0$  for  $s \geq t+N$ . At the beginning of period  $t$ , we know

$$(O_{t,t}, \dots, O_{t,t+N-1}).$$

**Example:** Assume that  $N = 2$ . At the beginning of period  $t$ ,  $O_{t,t} = D_{t-2,t} + D_{t-1,t}$ ,  $O_{t,t+1} = D_{t-1,t+1}$  and  $O_{t,t+2} = 0$ , while  $U_{t,t} = D_{t,t}$ ,  $U_{t,t+1} = D_{t,t+1} + D_{t+1,t+1}$ , and  $U_{t,t+2} = D_{t,t+2} + D_{t+1,t+2} + D_{t+2,t+2}$ . Notice that  $O_{t,s} + U_{t,s} = \sum_{j=0}^2 D_{s-j,s}$  for  $s \in \{t, t+1, t+2\}$ .  $D_{s-i,s} \equiv 0$  for  $i = 1, 2$  models the case of no advance demand information, while the case  $D_{s,s} \equiv 0$  models the case where all the demand information is obtained at least one period in advance.

We assume throughout that there is a centralized decision maker who has the opportunity to decide at the beginning of each period how much to order/produce and when and where to ship goods taking into account the advance demand information at hand. The objective is to establish an optimal control mechanism to minimize the expected discounted cost of managing the production/distribution system over a finite or infinite horizon. Since decisions are sequential and made under uncertainty, the problems will be formulated using dynamic programming.

### 3.2. Single location models

In this section, we consider a single location inventory model with fixed lead time  $L$  under advance demand information. At the beginning of each period the inventory manager has to decide whether or not to order and how much to order from its supplier in light of the advance demand information.

At the beginning of period  $t$ , the inventory manager knows

$I_t$ : inventory on hand

$B_t$ : number of backorders

$z_s$ : pipeline inventory  $s \in \{t - L, \dots, t - 1\}$

$O_{t,s}$ : observed part of demand for periods  $s \in \{t, \dots, t + N - 1\}$ .

We assume that the unsatisfied demands are backordered and satisfied as inventory is available. After observing the inventory on hand, the number of backorders, the pipeline inventory, and the advance demand information the manager places an order of size  $z_t \geq 0$ . This order will arrive at the beginning of period  $t + L$ . We assume that the cost of ordering  $z_t \geq 0$  units in period  $t$  is given by  $K_t \delta(z_t) + c_t z_t$  where  $K_t \geq 0$  is the set-up cost and  $\delta(z_t) = 1$  if  $z_t > 0$  and zero otherwise. This cost is realized whenever the order is placed. This assumption can be easily modified to incorporate other cases, including that where the cost is realized at the time of delivery. After the ordering decision is made, the demand vector  $D_t$  is realized. Demand is satisfied from on-hand inventory given priority to existing backorders, if any. At the end of each period, holding cost is charged based on the inventory on hand, if there is any. Otherwise, a penalty cost is charged based on the number of backorders.

The net inventory at the end of period  $t + L$  is given by

$$I_t + \sum_{s=t-L}^{t-1} z_s - B_t + z_t - D_t^L,$$

where  $D_t^L$  is the total demand realized during the periods  $t, t + 1, \dots, t + L$ . We use the term *protection period demand* to refer to the demand over these periods. Some authors follow a different convention and refer to  $D_t^L$  as the lead time demand. Notice that viewed from the beginning of period  $t$ , we can divide the protection period demand into the *observed* part

$$O_t^L \equiv \sum_{s=t}^{t+L} O_{t,s},$$

and the *unobserved* part

$$U_t^L \equiv \sum_{s=t}^{t+L} U_{t,s}.$$

This suggests a way to summarize the state space. Let

$x_t$  : modified inventory position before the ordering decision is made  $\equiv$

$$I_t + \sum_{s=t-L}^{t-1} z_s - B_t - O_t^L,$$

$y_t$  : modified inventory position after the ordering decision is made  $\equiv$

$$x_t + z_t$$

We use the term *modified* to distinguish the definition from the classical definition of inventory position, which does not subsume the observed part of the protection period demand. In addition to  $x_t$ , we have to keep track of the observations beyond the protection period,

$$O_t = (O_{t,t+L+1}, \dots, O_{t,t+N-1}).$$

To summarize, the state of the system is given by  $(x_t, O_t)$ . Notice that the vector  $O_t$  is meaningful only if  $N > L+1$ . Consequently, if  $N \leq L+1$  the state space is given solely by the modified inventory position.

The single period cost charged to period  $t$  is the discounted expected holding and penalty cost at the end of period  $t + L$ . This is given by

$$\tilde{G}_t(y_t) = \alpha^L E g_t(y_t - U_t^L),$$

where  $\alpha < 1$  is the discount factor,  $g_t(\cdot)$  is the holding and penalty cost function, and the expectation is taken with respect to the unobserved part of the protection period demand  $U_t^L$ . For each  $t$ , we assume that (i)  $g_t$  is convex, (ii)  $\lim_{|x| \rightarrow \infty} g(x) = \infty$  and (iii)  $E(D_{t,s})^\rho < \infty$  for some  $\rho > 1$ . These are classical assumptions in inventory literature. All these assumptions are satisfied when the holding and backorder penalty costs are linear, e.g., at rates  $h_t$  and  $p_t$  respectively.

After observing  $D_t = (D_{t,t}, \dots, D_{t,t+N})$  the *modified* inventory position is updated by

$$x_{t+1} = x_t + z_t - D_{t,t} - \sum_{s=t+1}^{t+L+1} D_{t,s} - O_{t,t+L+1} \quad (5.7)$$

and the vector of observed demand beyond the protection period by

$$O_{t+1} = (O_{t+1,t+L+2}, \dots, O_{t+1,t+N}) \quad (5.8)$$

where  $O_{t+1,s} = O_{t,s} + D_{t,s}$ .

A rigorous proof of the state space reduction is given in Özer [34].  $N = 0$  is the classical case studied extensively in inventory theory, see Scarf [37], Veinott [43], and Zheng [45].

For  $1 \leq N \leq L + 1$ , although there is non-trivial information about future demands, it is subsumed in the modified inventory position so *all* the classical results described for the case  $N = 0$  apply! In addition, if ordering takes place in a period then the order quantity is increasing in the observed protection period demand. Brown et al.[5] consider this case and restrict the information horizon to be, at most, the length of the protection period.

We let  $T$  denote the length of the horizon when it is finite. We assume that inventory leftovers at the end of the planning horizon are salvaged at unit rate  $c_{T+1}$ . Final backorders (if there are any) are satisfied by a final procurement at unit rate  $c_{T+1}$ . Our final assumption is that discounted set-up costs are non-increasing. More formally, we assume that  $\alpha K_{t+1} \leq K_t$  holds for all  $t$  where  $\alpha$  is the discount rate. None of these assumptions are stronger than the assumptions of classical inventory problems, see Scarf [37], Veinott [43].

### 3.2.1 Inventory problems with positive set-up costs.

This section focuses on problems with positive set up costs,  $K_t > 0$ , where the information horizon satisfies  $N > L + 1$ . We first characterize the form of optimal policies for the finite horizon case and then for the infinite horizon case. Recall that  $O_t$  is known at the beginning of period  $t$ . From now on we denote random variables or random vectors by lower case letters when their realization is known, e.g.,  $O_t = o_t$  when  $O_t$  is known.

The optimal cost-to-go function satisfies the dynamic program

$$J_t(x_t, o_t) = \min_{y_t \geq x_t} \{K_t \delta(y_t - x_t) + V_t(y_t, o_t)\} \quad (5.9)$$

where  $J_{T+1}(\cdot, \cdot) \equiv 0$ ,

$$V_t(y_t, o_t) = G_t(y_t) + \alpha E J_{t+1}(x_{t+1}, O_{t+1}), \quad (5.10)$$

and  $G_t(y) = (c_t - \alpha c_{t+1})y + \alpha^L E g_t(y - U_t^L)$ . The expectation in (5.10) is with respect to the vector  $D_t = (D_{t,t}, D_{t,t+1}, \dots, D_{t,t+N})$ . An intuitive explanation of this dynamic program is as follows: If the decision is to order  $y_t > x_t$  then we incur fixed and variable ordering costs plus cost of managing the system for a single period *plus* cost of managing this system starting from the next period to the end of the planning horizon  $T$ . Notice that we have subsumed the terminal condition into the dynamic program itself which results in a dynamic program with zero terminal cost. A formal construction of this DP can be found in Gallego and Özer [17].

Equation (5.9) can be expressed as

$$J_t(x_t, o_t) = V_t(x_t, o_t) + \min\{H_t(x_t, o_t), 0\}$$

where  $H_t(x_t, o_t) := K_t + \min_{y_t \geq x_t} V_t(y_t, o_t) - V_t(x_t, o_t)$ . If  $H_t(x_t, o_t) \leq 0$ , then it is optimal to order. On the other hand, if  $H_t(x_t, o_t) > 0$ , it is not optimal to order. If  $H_t(\cdot, o_t)$  has a *unique* sign change from  $-$  to  $+$  for every  $o_t$  then the policy has a simple form: an interval in which ordering is optimal followed by an interval in which ordering is not optimal. It can be shown, by contradiction and induction arguments and the definition of  $K$ -Convexity, that  $H_t(\cdot, o_t)$  has a unique sign change form  $-$  to  $+$ . The following result establishes the optimality of state dependent  $(s, S)$  policies.

**Theorem 5.1** *The following statements are true for any fixed vector  $o_t$ :*

- 1  $V_t(\cdot, o_t)$  is  $K_t$ -convex and  $\lim_{|x| \rightarrow \infty} V_t(x, o_t) = \infty$ .
- 2 An optimal policy is defined by a state dependent  $(s_t(o_t), S_t(o_t))$ -policy where

$$\begin{aligned} S_t(o_t) &= \min\{y : V_t(y, o_t) \leq V_t(x, o_t) \text{ for all } x\}, \\ s_t(o_t) &= \max\{x : H_t(x, o_t) \leq 0\}. \end{aligned}$$

- 3  $J_t(\cdot, o_t)$  is  $K_t$ -convex and  $\lim_{x \rightarrow \infty} J_t(x, o_t) = \infty$ ,  $\lim_{x \rightarrow -\infty} J_t(x, o_t) = V_t(s_t(o_t), o_t)$ .

Thus, it is optimal to order up to  $S_t(o_t)$  only if the modified inventory position is below  $s_t(o_t)$ . Notice that these critical levels depend on the observation beyond the lead times.

**3.2.2 Stationary problems.** We refer to an inventory problem as stationary if the demand and the cost parameters are stationary, i.e.  $c_t = c$ ,  $g_t = g$ ,  $\alpha_t = \alpha$  and  $K_t = K$  and we also drop the subscript from single period cost function  $G$ . Let us define for stationary problems

$$\begin{aligned} S^* &= \min\{y : G(y) \leq G(x) \text{ for all } x\} \\ s^* &= \max\{y \leq S^* : G(y) \geq K + G(S^*)\} \\ \bar{S} &= \min\{y > S^* : G(y) > G(S^*) + \alpha K\} \end{aligned}$$

The pair  $(s^*, S^*)$  is a myopic policy for the positive set-up cost case. Such a policy ignores the impact of current decisions to future and also

does not depend on advance demand information. Myopic policies for non-stationary problems are defined similarly but they are time dependent. The points above exist since  $G$  is convex with respect to  $y$  and  $\lim_{|y| \rightarrow \infty} G(y) = \infty$ .

The following Lemma shows that optimal policies are, in a sense, bounded by myopic policies.

**Lemma 5.2** *For all  $t$  and any fixed vector  $o_t$ ,  $S^* \leq S_t(o_t) \leq \bar{S}$  and  $s^* \leq s_t(o_t)$ .*

We remark that our proofs do not require the underlying functions to be continuous and differentiable. Our arguments are based on the study of first differences and allow us to cover demand processes with integer domains.

The next Theorem provides a horizon result that sheds more light on the structure of optimal policies under advance demand information.

**Theorem 5.3** *For finite horizon stationary problems, if  $(\bar{S} - s^*) \leq o_{t,t+L+1}$ , then  $S_t(o_t) = S^*$ .*

This result shows that once the observed demand for period  $t + L + 1$  exceeds  $\bar{S} - s^*$ , the myopic order-up-to level is optimal for the stationary positive set-up cost problems. The threshold level is a function of the lower bound for the reorder point and the upper bound for the order-up-to level. Tighter bounds result in a lower threshold level. As the set-up cost increases the observed demands for the immediate period beyond the protection period need to be higher for the horizon result to hold. This result has both managerial and computational implications. Management can ignore advance demand information beyond period  $t + L + 1$  if the observed demand for period  $t + L + 1$  is sufficiently high. In particular, management should concentrate on ordering, if needed, to satisfy the demand for period  $t + L$ , knowing that a new order will be placed in period  $t + 1$ . The horizon result limits the need to search for state dependent policies, when the observed demand for period  $t + L + 1$  is sufficiently large, making it easier to compute optimal policies.

**Theorem 5.4** *The results obtained for the stationary finite horizon problem with positive set-up cost also hold for stationary infinite horizon problems.*

**3.2.3 Inventory problems with zero set-up costs.** The functional equation for the zero set-up cost case is given by equation (5.9) with  $K_t = 0$  for all  $t$ . Using similar arguments as in the case of positive set up cost, we obtain the following results for the finite horizon zero set-up cost case.

**Theorem 5.5** *The following statements are true for any vector  $o_t$ :*

- 1  $V_t(\cdot, o_t)$  is convex and  $\lim_{|x| \rightarrow \infty} V_t(x, o_t) = \infty$ .
- 2 An optimal ordering policy is a state dependent base-stock policy where the order-up-to level is given by the smallest minimizer of  $V_t(\cdot, o_t)$ , i.e.

$$y_t(o_t) = \min \left\{ y : V_t(y, o_t) = \min_x V_t(x, o_t) \right\}. \quad (5.11)$$

- 3  $J_t(\cdot, o_t)$  is increasing convex.
- 4  $V_t(x, o_t)$  has decreasing differences<sup>4</sup> in  $(x, o_t)$ .
- 5  $y_t(o_t)$  is increasing in  $o_t$ .
- 6  $J_t(x, o_t)$  has decreasing differences in  $(x, o_t)$ .

This indicates that an optimal policy is to order whenever the modified inventory position falls below a state dependent base stock level. The fifth statement shows that systems maintain higher order-up-to levels, hence higher average inventory levels, as the observed demand beyond the protection period increases.

**3.2.4 Stationary policies.** The following result is necessary to establish optimal policies for the stationarity zero set-up cost infinite horizon case.

**Lemma 5.6** *For all  $t$  and any vector  $(x, o)$ ,  $V_{t-1}(x, o) \geq V_t(x, o)$ ,  $y_{t-1}(o) \leq y_t(o)$ ,  $J_{t-1}(x, o) \geq J_t(x, o)$ .*

A myopic policy ignores the effect of upcoming periods and focuses on minimizing the expected cost for the current period. Thus a myopic policy for the zero set-up cost case is any minimizer of  $G_t$ , i.e., any  $y \in (y_{min}^*, y_{max}^*)$  where

$$y_{min}^* = \min \left\{ y : G_t(y) = \min_x G_t(x) \right\},$$

$$y_{max}^* = \max \left\{ y : G_t(y) = \min_x G_t(x) \right\}.$$

Notice that the range collapses into a unique point when  $G_t(\cdot)$  is strictly convex.

**Theorem 5.7** *For a stationary problem any base stock policy, where the base stock level  $y^* \in (y_{min}^*, y_{max}^*)$ , is optimal for the finite horizon problem.*

This result shows that information beyond the protection period does not affect the order-up-to level when we assume stationary costs and demand distributions. Intuitively, it makes sense to order only to cover for the protection period demand in the absence of fixed costs. This result significantly reduces the computational effort since the state space collapses to a single dimension. It also implies that management does not need to obtain advance demand information beyond the protection period for inventory control purposes. As with the case of positive set up cost the results carry over to the infinite horizon.

**3.2.5 Numerical study.** We use a backward induction algorithm to solve the functional equation (5.9). For the purpose of our numerical study we assume  $L = 0$  and  $N = 2$ . This is the simplest case for which the problem is non-trivial and is general enough to capture the main ideas. We model  $D_{t,t+i}$  as Poisson with parameter  $\lambda_i, i = 0, 1, 2$ . Notice that  $o_t = D_{t-1,t+1}$ .

Figure 5.1 depict the relationship between  $x_t$  and  $y_t$  with respect to  $o_t = D_{t-1,t+1}$ , the observed demand information beyond the protection

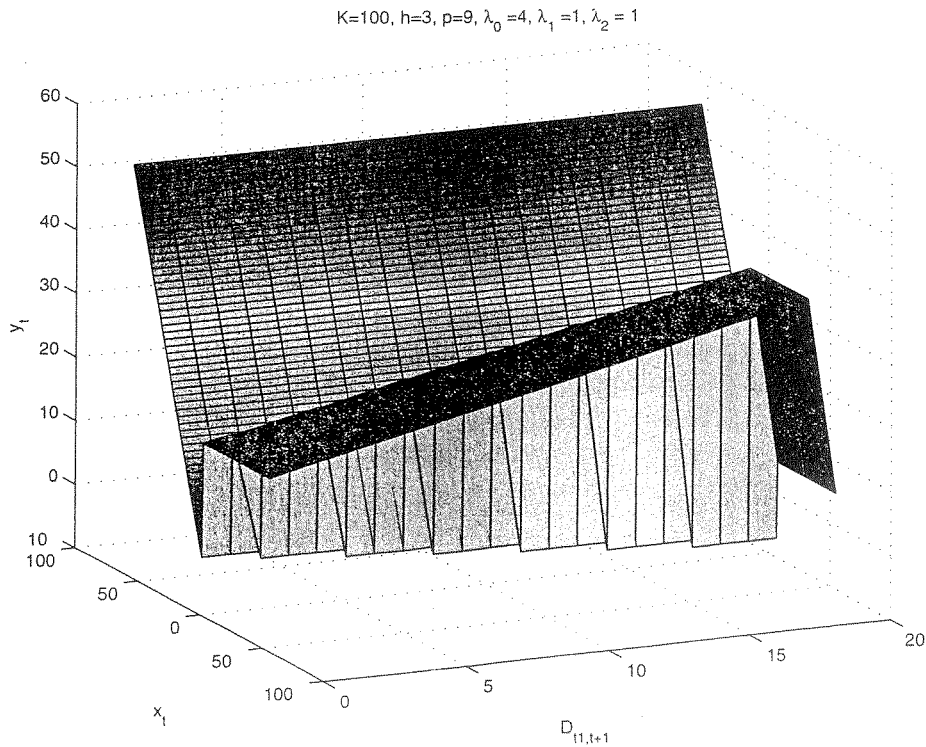


Figure 5.1. Positive set-up cost

period. Notice that order-up-to level increases as the level of observed demand increases for large set-up costs. We find counter examples, however, which show that this monotonistic behavior is not a general property. On the other hand, our extensive experiments indicate that reorder point  $s_t(D_{t-1,t+1})$  decreases as  $D_{t-1,t+1}$  increases. We found this observation surprising, because intuition suggests that the reorder point,  $s_t(D_{t-1,t+1})$ , should be increasing in  $D_{t-1,t+1}$  making it more likely to place an order to cope with large observed demands. Careful thought, however, reveals a more complete story. First, notice that if  $x_t$  is not too low, the holding and penalty cost of not ordering may be lower than the cost of ordering and carrying  $D_{t-1,t+1}$  for one period. This suggests that at high values of  $D_{t-1,t+1}$ , it may be better to incur a shortage cost now rather than to place an order and carry inventory for the next period. On the other hand, for sufficiently low values of  $x_t$  and very high values of  $D_{t-1,t+1}$  it is best to place two consecutive orders, which is shown in Theorem 5.3. In this case, it is optimal to raise the modified inventory position of the first order to minimize current costs, i.e.,  $S_t(D_{t-1,t+1}) = S^*$  when  $D_{t-1,t+1} \geq \bar{S} - s^*$ . A sharp decline of order-up-to-level in Figure 5.1 depicts this result.

We now illustrate how our model quantifies the trade off between the benefits of advance demand information and the cost of implementing a pricing strategy that induces advance bookings. Recall that the total  $s$  demand for period  $s$  is given by  $\mathcal{D}_s = D_{s-2,s} + D_{s-1,s} + D_{s,s}$ . In Table 5.1, we fix the expected total demand for a period to be 6(=  $\lambda_0 + \lambda_1 + \lambda_2$ ) and increase  $\lambda_2$  while decreasing  $\lambda_0$  (This allows us to model when the inventory manager has more advance demand information). Assume that a brand manager is trying to acquire advance demand information through pricing strategies. She is willing to reduce the price of the product if customers are willing to book early. Strategies one

Table 5.1. Optimal  $(s, S)$  levels with respect to advance demand information

$K = 100, h = 1, p = 9, s_t(D_{t-1,t+1}), S_t(D_{t-1,t+1})$ for $D_{t-1,t+1} \in \{0, \dots, 15\}, T = 12$										
#	$(\lambda_0, \lambda_1, \lambda_2)$		0	1	...	14	15	$(s^*, S^*, S)$	$J_1(0, 0)$	$J_1(0, 10)$
1	(5,1,0)	$S(\cdot)$	35	36	...	49	50	(-7,8,110)	418.81	427.80
		$s(\cdot)$	2	2	...	0	0			
2	(4,1,1)	$S(\cdot)$	33	34	...	47	48	(-8,7,108)	415.81	420.12
		$s(\cdot)$	1	1	...	-1	-1			
3	(1,1,4)	$S(\cdot)$	27	28	...	41	42	(-11,2,104)	379.77	386.85
		$s(\cdot)$	-2	-2	...	-3	-3			
4	(0,1,5)	$S(\cdot)$	25	26	...	39	40	(-12,0,100)	353.91	375.67
		$s(\cdot)$	-3	-3	...	-4	-4			

through four in Table 5.1 model different levels of aggressiveness in the pricing strategy to induce advance bookings. The last two columns in Table 5.1 show the reduction in costs for two initial states. Notice that expected cost decreases as more and more customers are induced to book early. More examples can be found in Gallego and Özer [17]. It is also evident from this table that the order-up-to level and the reorder point decrease as more customers place advance orders, suggesting a reduction in average inventory level.

### 3.3. Multi-stage serial systems

In this section, we incorporate advance demand information for a multi-stage serial system and establish that state dependent base stock policies are optimal. Consider a production/distribution system with  $J$  stages. External demand occurs only at stage  $J$ . Stage  $j \geq 2$  satisfies its requirements from Stage  $j - 1$ . The first stage orders from an outside supplier with ample stock. Shipments arrive after exogenous, stage specific, lead times. Replenishment decisions are centralized and based on system wide inventory information. The objective is to minimize the expected discounted cost of managing the system over a finite horizon. A two stage serial system can be interpreted as a manufacturer and a retailer. The retailer procures from the manufacturer to satisfy an uncertain demand process. The manufacturer satisfies its own requirement from an outside supplier. A centralized decision maker has to decide when and how much to procure from the outside supplier and when and how much to ship to the retailer to minimize the holding and penalty cost of managing the system over a finite horizon in light of advance demand information. The infinite horizon problem is essentially a limiting case of the finite horizon problem. More details can be found in Gallego and Özer [18].

Clark and Scarf [11] show that the problem without advance demand information decomposes into single stage problems and prove the optimality of base stock policies. Federgruen and Zipkin [13] extend these results for stationary infinite horizon problems. Chen and Zheng [9] establish lower bounds on the cost of managing the infinite horizon inventory problems for the average cost criteria and construct feasible policies that achieve these lower bounds. Chen and Song [8] is the only paper to establish the form of optimal policies for the infinite horizon case with non-stationary demands. In their model, demand is governed by a Markov modulated Poisson process. They prove the optimality of state dependent echelon base stock policies and provide an algorithm to calculate these policies. Rosling [36] shows the equivalence of series

and assembly systems under a mild assumption—long run balance—on the initial stock levels. We refer the reader to Federgruen [12] for a further discussion on the relation among series, distribution and assembly systems.

**3.3.1 Impact of advance demand information.** The demand process is as described in section 3.1. At the beginning of each period, after receiving previously placed orders and/or scheduled shipments, the decision maker decides how much to order, say  $z_1 \geq 0$ , from an outside supplier and how much to ship, say  $z_j \geq 0$ , to Stage  $j \in \{2, \dots, J\}$  from Stage  $j - 1$ . A linear ordering/shipping cost  $\sum_{j=1}^J c_{jt} z_j$  is charged to period  $t$ . An order from the outside supplier placed at the beginning of period  $t$  arrives at Stage 1 at the beginning of period  $t + L_1$ . Similarly, a shipment to Stage  $j \in \{2, \dots, J\}$  placed at the beginning of period  $t$  arrives at the beginning of period  $t + L_j$ .

At the beginning of period  $t$ , in addition to the on hand inventory  $I_{jt}$ , at stage  $j = 1, \dots, J$ , and the backorders  $B_t$  at Stage  $J$ , the decision maker also knows

$$\begin{aligned} \vec{O}_t &= (O_{t,t}, \dots, O_{t,t+N-1}) \\ \vec{z}_{jt} &= (z_{j1}, \dots, z_{jL'_j}) \text{ trans-shipments to Stage } j \end{aligned}$$

for all  $j \in \{1, \dots, J\}$  and  $L'_j \equiv L_j - 1$ . Here  $\vec{O}_t$  is the observed part of the demand over periods  $\{t, \dots, t + N - 1\}$ ,  $z_{js}$  is the shipment dispatched from Stage  $j - 1$  to Stage  $j$  at the beginning of period  $t - s$ . The echelon net inventory includes shipments placed a lead time earlier and received at the beginning of the period. Under this convention,  $\vec{z}_{jt}$  is irrelevant whenever  $L_j \in \{0, 1\}$ . We define

$$\begin{aligned} \hat{x}_{jt} &= \text{echelon net inventory at Stage } j \\ x_{jt} &= \text{echelon inventory position at Stage } j \\ x_{jt}^{L_j} &= \text{modified echelon inventory position at Stage } j \end{aligned}$$

the echelon *net* inventory  $\hat{x}_{jt}$  to be the inventory on hand at the beginning of period  $t$  at stages  $\{j, j + 1, \dots, J\}$  plus the inventory in transit between these stages minus the backorder at Stage  $J$ . The echelon inventory *position* at Stage  $j$  is the echelon net inventory at stage  $j$  plus inventories in transit to Stage  $j$ . Notice that  $\hat{x}_{Jt} = I_{Jt} - B_t$ ,  $x_{jt} = \hat{x}_{jt} + \sum_{l=1}^{L'_j} z_{jl}$  and  $\hat{x}_{jt} = x_{j+1t} + I_{jt}$  for  $j \in \{1, \dots, J - 1\}$ . Finally, in order to reduce the dimension of the state space we define, for each stage, the *modified* echelon inventory position as the echelon inventory position minus the observed part of the protection period demand

corresponding to that stage, i.e.,

$$x_{jt}^{L_j} = x_{jt} - O_t^{L_j} \quad \text{for } j \in \{1, \dots, J\},$$

where  $O_t^{L_j} \equiv \sum_{s=t}^{t+L_j} O_{t,s}$ . For convenience we also define  $U_t^{L_j} = \sum_{s=t}^{t+L_j} U_{t,s}$ .

For each period  $t$  the system has the following cost parameters:

$$\begin{aligned} p_t &= \text{penalty cost (at Stage } J) \text{ per unit,} \\ h'_{jt} &= \text{local inventory holding cost at Stage } j \text{ per unit,} \\ h_{jt} &= \text{echelon holding cost at Stage } j \text{ per unit} \\ &= h'_{jt} - h'_{j-1,t} \text{ for } j \in \{2, \dots, J\} \text{ and } h_{1t} = h'_{1t}, \\ c_{jt} &= \text{shipment cost per unit in period } t. \end{aligned}$$

The holding and penalty costs for a period are based on the inventory levels at the end of the period. We assume that the echelon holding costs are strictly positive.

The period  $t$  holding and penalty cost is given by

$$\begin{aligned} & \sum_{j=1}^{J-1} h'_{jt} [\hat{x}_{j,t+1} - \hat{x}_{j+1,t+1}] + h'_{Jt} [\hat{x}_{J,t+1}]^+ + p_t [\hat{x}_{J,t+1}]^- \\ &= \sum_{j=1}^J h_{jt} \hat{x}_{j,t+1} + [p_t + h'_{Jt}] [\hat{x}_{J,t+1}]^- \end{aligned}$$

where  $[x]^+ = \max(x, 0)$  and  $[x]^- = \max(-x, 0)$ . Notice that shipments in transit to Stage  $j+1$  are charged at Stage  $j$ 's holding cost rate. At the end of period  $t$  the updates are given by

$$\begin{aligned} \bar{O}_{t+1} &= (O_{t+1,t+1}, \dots, O_{t+1,t+N}) \\ \bar{z}_{j,t+1} &= (z_j, z_{j1}, \dots, z_{jL'_j-1}) \\ \hat{x}_{j,t+1} &= \hat{x}_{jt} + z_{jL'_j} - o_{t,t} - D_{t,t} \\ x_{j,t+1} &= x_{jt} + z_j - o_{t,t} - D_{t,t} \\ x_{j,t+1}^{L_j} &= x_{jt}^{L_j} + z_j - \sum_{s=t}^{t+L_j+1} D_{t,s} - O_{t,t+L_j+1} \end{aligned}$$

For finite horizon problems, we assume a linear terminal condition of the form  $-\sum_{j=1}^J c_j x_j$ . The economic interpretation is that of a salvage value if  $x_j$  is positive, and an acquisition cost if  $x_j$  is negative. Notice that the  $c_j$ 's are actually echelon costs so the terminal condition makes economic sense. To our knowledge, all other papers in the literature charge zero

terminal costs. Linear terminal costs are more realistic and allows us to show that myopic policies are optimal for finite horizon problems with stationary costs and demand distributions, a result that fails to hold under zero terminal costs.

To simplify the exposition of the results we assume  $J = 2$ . At the end of this section we discuss briefly the extension for more than two stages. The problem in this case is to manage the two stage series inventory system for periods  $\{t, \dots, T\}$ . As in single location problem, we use dynamic programming to solve the multi-locations in series. The last order for Stage 1 is dispatched at the beginning of period  $T$ , and arrives to Stage 1 at the beginning of period  $T + L_1$ . The last shipment to Stage 2, initiated at the beginning of period  $T + L_1$ , arrives at Stage 2 at the beginning of period  $T + L_1 + L_2$ . Thus, for  $t > T + L_1$  there will not be any inventory on order between the supplier and the first stage. Likewise, all the shipments would have arrived to the second stage by the beginning of period  $T + L_1 + L_2$ . We assume that cost continues to accrue up to period  $T + L_1 + L_2$ . All holding and penalty costs after time  $T + L_1 + L_2 + 1$  are assumed to be zero. Also,  $\vec{O}_{T+L_1+L_2+1} = 0$ . This means that we take advance information up to the period  $T + L_1 + L_2$  only. The state space will be initially defined by  $(\hat{x}_{1t}, \vec{z}_{1t}, \hat{x}_{2t}, \vec{z}_{2t}, \vec{O}_t)$ . As in the single location case we denote random vectors by lower case when their realization is known, e.g.  $\vec{O}_t = \vec{o}_t$  when  $\vec{O}_t$  is known.

The optimal cost-to-go starting from period  $t$  is given by,

$$\begin{aligned} & \hat{J}_t(\hat{x}_{1t}, \vec{z}_{1t}, \hat{x}_{2t}, \vec{z}_{2t}, \vec{o}_t) \\ &= \min_{(z_1, z_2) \in \mathcal{A}'} \{c_{1t}z_1 + c_{2t}z_2 + h_{1t}E\hat{x}_{1,t+1} + Eg_t(\hat{x}_{2,t+1}) \\ & \quad + \alpha E\hat{J}_{t+1}(\hat{x}_{1,t+1}, \vec{z}_{1,t+1}, \hat{x}_{2,t+1}, \vec{z}_{2,t+1}, \vec{O}_{t+1})\} \end{aligned} \quad (5.12)$$

where  $\hat{J}_{T+L_1+L_2+1}(\hat{x}_1, \cdot, \hat{x}_2, \cdot, \cdot) \equiv -c_1\hat{x}_1 - c_2\hat{x}_2$ ,  $g_t(x) = h_{2t}[x] + (p_t + h'_{2t})[x]^-$ , and  $\mathcal{A}' = \{(z_1, z_2) \in Z^2 : z_1 \geq 0, z_2 \geq 0, \text{ and } x_{2t} + z_2 \leq \hat{x}_{1t}\}$ . Notice that shipment constraint  $x_{2t} + z_2 \leq \hat{x}_{1t}$  is equivalent to  $z_2 \leq I_{1t}$ , meaning that shipments to Stage 2 are bounded by the inventory on hand at Stage 1. Notice also that  $g_t$  is a convex function and that  $\lim_{|x| \rightarrow \infty} g_t(x) = \infty$ .

The dynamic program can be decomposed into two simpler problems. First, by virtue of  $x_{2t} = \hat{x}_{2t} + \sum_{l=1}^{L'_2} z_{2l}$ , and standard cost accounting manipulations, the state space reduces to  $(\hat{x}_{1t}, \vec{z}_{1t}, x_{2t}, \vec{o}_t)$ , so we don't need to keep track of the vector  $\vec{z}_{2t}$  of shipments from Stage 1 to Stage 2. This yields a dynamic program with cost-to-go  $J_t(\hat{x}_{1t}, \vec{z}_{1t}, x_{2t}, \vec{o}_t)$ . Second, this program can be decomposed into two simpler dynamic programs:

$$J_t(\hat{x}_{1t}, \vec{z}_{1t}, x_{2t}, \vec{o}_t) = \hat{V}_t^1(\hat{x}_{1t}, \vec{z}_{1t}, \vec{o}_t) + \tilde{V}_t^2(x_{2t}, \vec{o}_t).$$

We will not define these intermediate dynamic programs here since we can further reduce the dimension of each of these programs by using the *modified* inventory position concept.<sup>5</sup>

**Theorem 5.8** *The dynamic program for the series system decomposes into two simpler dynamic programs given by equation (5.13) and (5.14) (defined below), which can be interpreted as single location problems under advance demand information.*

The dynamic program for stage two is given by

$$V_t^2(x_{2t}^{L_2}, o_t^2) = -c_{2t}x_{2t}^{L_2} + \min_{x_{2t}^{L_2} \leq y} \{H_t(y, o_t^2)\} \quad (5.13)$$

where  $H_t(y, o_t^2) = c_{2t}y + G_t(y) + \alpha EV_{t+1}^2(x_{2,t+1}^{L_2}, O_{t+1}^2)$ , this function is convex and goes to infinity as  $|y|$  tends to infinity,  $V_{T+L_1+L_2+1}^2(x^{L_2}, \cdot) \equiv -c_{2t}x^{L_2}$  and  $o_t^2 \equiv (o_{t,t+L_2+1}, \dots, o_{t,t+N-1})$ . Let  $y_{2t}(o_t^2)$  denote the smallest minimizer of  $H_t(\cdot, o_t^2)$ .

Observe that the above dynamic program is similar to single location problem for which the optimal policy is a state dependent base stock policy. Under this policy the manager orders up to  $y_{2t}(o_t^2)$  if the modified inventory position is below this level to achieve the minimum of function  $H_t(\cdot, o_t^2)$ . In a two stage series system, however, the ordering decision will be constrained by the available inventory at the first stage. Hence, if the first stage turns out to be a bottle neck, it should bare the consequences. Let  $IP_t(x, o_t^2) = H_t(\min\{x, y_{2t}(o_t^2)\}, o_t^2) - H_t(y_{2t}(o_t^2), o_t^2)$ . This implicit cost function appears in the dynamic program for stage one. Let

$$V_t^1(x_{1t}^{L_1}, o_t^1) = -c_{1t}x_{1t}^{L_1} + \min_{y \geq x_{1t}^{L_1}} \{c_{1t}y + C_t(y, o_t^1) + \alpha EV_{t+1}^1(x_{1,t+1}^{L_1}, O_{t+1}^1)\} \quad (5.14)$$

where  $V_{T+L_1+1}^1(x_{1t}^{L_1}, \cdot) \equiv -c_{1t}x_{1t}^{L_1}$ ,  $o_t^1 \equiv (o_{t,t+L_1+1}, \dots, o_{t,t+N-1})$ ,

$$C_t(y, o_t^1) = \alpha^{L_1} E \hat{C}_{t+L_1}(y - U_t^{L_1}, O_{t+L_1}^1), \quad \text{and}$$

$$\hat{C}_t(y, o_t^1) = h_{1t}y + \alpha EIP_{t+1}(y, O_{t+1}^1).$$

Let  $y_{1t}(o_t^1)$  denote the smallest minimizer of the function inside the  $\{c_{1t}y + C_t(y, o_t^1) + \alpha EV_{t+1}^1(x_{1,t+1}^{L_1}, O_{t+1}^1)\}$ . To summarize, the problem of finding an optimal policy for a series system under advance demand information reduces to solving two simpler, single stage, dynamic programs. The state space for these programs is  $1 + (N - L_j - 1)^+$  for  $j = 1, 2$ .

**Theorem 5.9** *An echelon state dependent base stock policy is optimal for a two stage system in series. In particular, an optimal base stock level for stage  $j$  at time  $t$  is given by  $y_{jt}(\sigma_t^j)$  for  $j = 1, 2$ .*

The argument to establish the optimality for  $J > 2$  stages requires a recursive application of the decomposition argument until system is decomposed into single stage problems. Then we use the modified inventory position to further reduce the state space for each of the single stage problems.

**3.3.2 Myopic policies.** In this section, we construct myopic policies for the two-stage serial system. We assume that costs are stationary hence we drop the  $t$  from cost parameters, for example  $h_{1t} = h_1$  for all  $t$ . We further assume that  $D_t$  is a stationary vector.

Let  $y_2^m$  be the smallest minimizer of

$$\mathcal{L}_2(y) = (1 - \alpha)c_2y + G(y).$$

Notice that  $\mathcal{L}_2$  is convex and  $\lim_{|y| \rightarrow \infty} \mathcal{L}_2(y) = \infty$ , so  $y_2^m$  is finite. We refer to the policy that orders up to the base stock level  $y_2^m$  as the myopic policy for Stage 2.

Next, we define the myopic implicit penalty cost function and related cost functions

$$\begin{aligned} IP^m(x) &= \mathcal{L}_2(\min\{y_2^m, x\}) - \mathcal{L}_2(y_2^m) \\ C^m(x) &= \alpha^{L_1} E[h_1(x - U_t^{L_1}) + \alpha IP^m(x - U_t^{L_1})] \\ \mathcal{L}_1(y) &= (1 - \alpha)c_1y + C^m(y). \end{aligned}$$

Let  $y_1^m$  be the smallest minimizer of  $\mathcal{L}_1$ . Notice that  $\mathcal{L}_1$  is convex and  $\lim_{|y| \rightarrow \infty} \mathcal{L}_1(y) = \infty$ , so  $y_1^m$  is also finite. We refer to the policy that orders up to the base stock level  $y_1^m$  as the myopic policy for Stage 1. Finally, we refer to the policy that orders up to  $y_1^m$  for Stage 1 and up to  $y_2^m$  for Stage 2 as the myopic policy.

**Theorem 5.10** *Under stationary demand and cost parameters the myopic policy is optimal for finite horizon problems.*

**3.3.3 Numerical study.** We assume for our numerical study that  $L_1 = L_2 = 1$  and that  $N = 3$ . Recall that the dimension of the state space is for Stage  $j$  is  $1 + (N - L_j - 1)^+$ . Consequently, the state space is two dimensional for both stages. We will assume that  $D_{t,t+i}$  is Poisson with parameter  $\lambda_{ti} = w_i \Lambda_t$ , where  $w_i \geq 0$ , and  $\sum_{i=0}^3 w_i = 1$ . Hence  $\Lambda$ .

Table 5.2. Optimal echelon base stock levels ( $\Lambda_t = 3$ )

$h_1 = 1, h_2 = 3, c_1 = 10, c_2 = 30, p = 19$					
No.	$(w_0, w_1, w_2, w_3)$	$(y_{1t}, y_{2t})$	No.	$(w_0, w_1, w_2, w_3)$	$(y_{1t}, y_{2t})$
1	(1.0, 0.0, 0.0, 0.0)	(15,8)	4	(0.8, 0.2, 0.0, 0.0)	(15,7)
2	(0.4, 0.3, 0.2, 0.1)	(11,4)	5	(0.8, 0.0, 0.2, 0.0)	(14,7)
3	(0.1, 0.2, 0.3, 0.4)	(9,2)	6	(0.8, 0.0, 0.0, 0.2)	(13,7)

is average number of customers placing orders at time  $t$ , of which, on average,  $\lambda_{ti}$  place their orders to be delivered  $i$  periods later. By changing the weights,  $w_i$ , we can model the degree to which customers place orders in advance of their needs. Notice also that  $o_t^1$  and  $o_t^2$  are scalars.

The first experiment in Table 5.2 corresponds to customers requiring immediate delivery. The next two experiments represent the case where the manager obtains more advance demand information perhaps by inducing customers to place orders for future periods. The echelon base stock levels for both stages decrease as more of the demand is known in advance. Experiment No. 4-6 yield similar observations. Figure 5.2, exhibits the optimal echelon base stock levels through each period for a stationary demand process where the mean is  $\Lambda_t = 2$  for

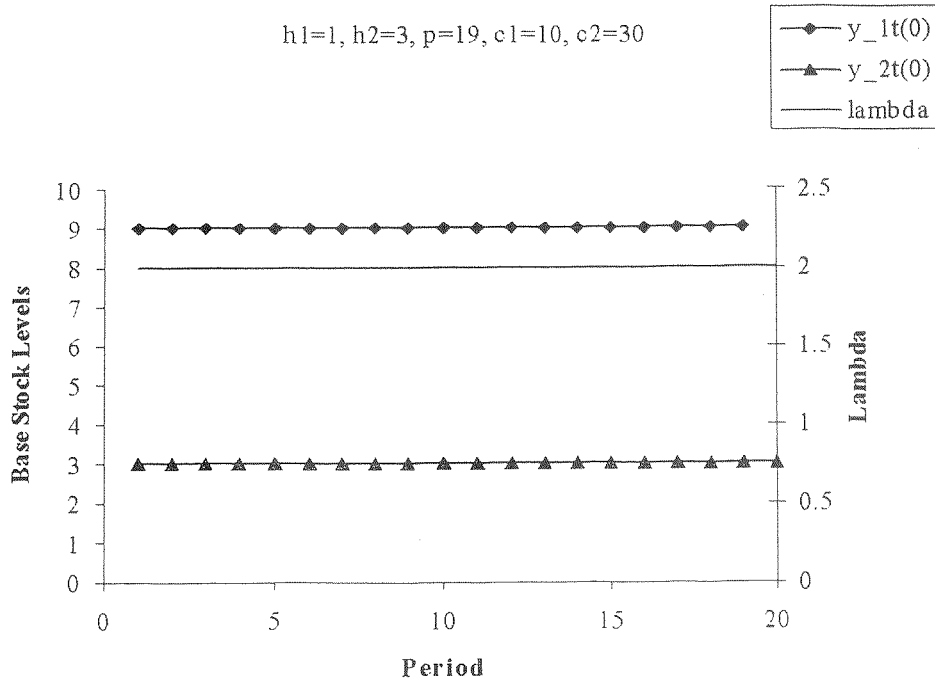


Figure 5.2. Stationary demand case

all  $t = \{1, \dots, 20\}$ . We observe that the myopic policies are stationary and optimal. Similarly, Figure 5.3 exhibits the optimal echelon base stock levels for a ramp-up demand process. We observe the optimality of myopic base stock levels which are non-decreasing, see also Table 5.3. This result can also be established analytically. Thus, advance demand information beyond the protection period has no operational value for both the stationary and the ramp up demand process.

### 3.4. One-warehouse multi-retailer systems

In this section, we study a periodic-review distribution system consisting of a central depot and  $J$  retailers under advance demand information

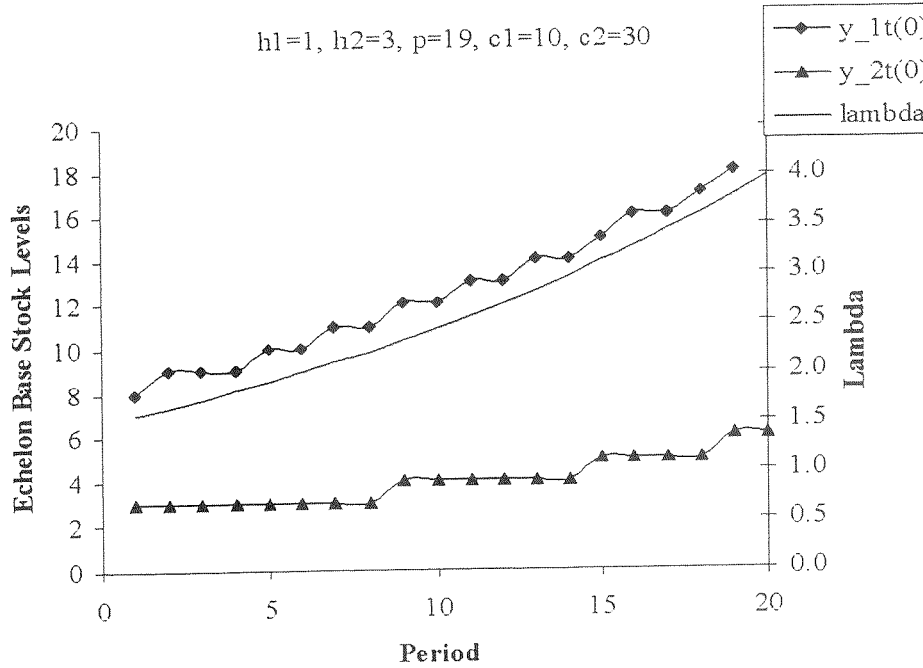


Figure 5.3. Ramp-up demand case

Table 5.3. Optimal echelon base stock levels

$D_{t-1,t+1}$	Stationary Demand Case							Ramp-Up Demand Case						
	0	1	2	3	4	5	$y_i^m$	0	1	2	3	4	5	$y_i^m$
$y_{1t}(D_{t-1,t+1})$	9	9	9	9	9	9	9	8	8	8	8	8	8	8
$y_{2t}(D_{t-1,t+1})$	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Note that  $O_t^1 = O_t^2 = D_{t-1,t+1}$  since  $L_1 = L_2 = 1$

and centralized control. We assume first that the depot is a coordination and re-packaging center so it does not hold any inventory. The uncertain demand is satisfied through the retailers and each retailer replenishes their inventory through a central depot. The depot satisfies its own requirement from an outside supplier with ample stock. Orders placed by the depot arrive after an exogenously specified fixed lead time  $L$ . Shipments to the retailers arrive after an exogenously specified fixed lead time  $l$ . Customers are satisfied from on hand inventory at the retailers and unsatisfied orders are backlogged. In section 3.4.3, we discuss briefly how to extend the results for systems where the central depot is allowed to carry inventories.

If holding inventory at the warehouse is not allowed, then the decision maker has to decide (i) whether or not to place an order from an outside supplier, and (ii) how to allocate the incoming order to the retailers to minimize the expected holding and shortage cost over a finite horizon. If central inventory is allowed, the decision maker must decide (i) whether or not to order, (ii) how much to withdraw from the warehouse and (iii) how to allocate the withdrawn amount to the retailers. We remark that the structure, if any, of optimal policies for distribution systems is unknown even in the absence of advance demand information. We, therefore, use a relaxation method to simplify the problem and use the solution to the relaxed problem to develop a heuristic that incorporates advance demand information. Our numerical study suggests that the heuristic performs well.

The distribution system described above can also be interpreted as a multi-item production/distribution system with a common intermediate product. In this interpretation, the depot represents the differentiation point. During the first phase of the production period,  $L$ , a common batch is produced. At the end of this period the manager has to decide how much of each item to produce from the batch that has just arrived. This interpretation forms the basis of postponement strategies, see for example Lee et al. [29], Aviv and Federgruen [2]. Through a numerical example we illustrate that advance demand information increases the benefit gained through postponement strategies and that the benefit obtained through advance demand information may far exceed the benefit of postponement strategies.

**3.4.1 Warehouse as coordination center.** During time period  $t$  at each retailer  $j$  we observe the demand vector

$$d_t^j = (d_{t,t}^j, \dots, d_{t,t+N_j}^j)$$

where  $d_{t,s}^j$  represents the demand for period  $s$  observed during period  $t$  at retailer  $j$ . Customers place order at retailer  $j \in \{1, \dots, J\}$  for future periods  $s \in \{t, \dots, t+N_j\}$  where  $N_j < \infty$  is the length of the *information horizon* for each retailer.

Demand at each retailer is satisfied from on-hand inventory, if any. Unsatisfied demand is backlogged. At the beginning of each period  $t$ , the manager reviews the on hand inventory, the pipeline inventory, backorders and the advance demand information at each retailer. He/she decides whether or not to place an order, say  $w_t \geq 0$ , from an outside supplier. The orders arrive after a positive fixed lead time,  $L \geq 0$ . Simultaneously, the orders placed in period  $t-L$  arrive to depot at the beginning of period  $t$ . The decision maker also decides how to allocate this batch to retailers, say  $z_t^j \geq 0$ . Clearly, we want total shipments to the retailers equal to the incoming orders to the depot,  $\sum_{j=1}^J z_t^j = w_{t-L}$ . Shipments to retailers arrive after a positive lead time,  $l \geq 0$ . A linear shipping cost of  $c_{0t}w_t + \sum_{j=1}^J c_t^j z_t^j$  is charged to period  $t$  where  $c_t^j$  is the variable shipping cost rate for retailer  $j \in 1, \dots, J$  and  $c_{0t}$  is the variable ordering cost rate from the outside supplier. Demands and costs are not necessarily stationary. The aim is to minimize the expected cost of managing this system over a finite horizon.

At the beginning of each time period  $t$ , the demand to be realized at retailer  $j$  during a future period  $s \in \{t, \dots, t+N_j-1\}$  is equal to the sum of the part that is *observed* and known to the decision maker  $\sigma_{t,s}^j = \sum_{r=s-N_j}^{t-1} d_{r,s}^j$ , and the part that is *unobserved* and unknown to the decision maker  $u_{t,s}^j = \sum_{r=t}^s d_{r,s}^j$ . Hence, at the beginning of period  $t$ , the decision maker knows  $I_t^j$  on hand inventory,  $B_t^j$  backorders,  $\sigma_{t,s}^j$  cumulative *observed* part of the demand for periods  $s \in \{t, t+1, \dots, t+N_j-1\}$  for each retailer  $j = 1, \dots, J$ .

After reviewing this information and receiving the order  $w_{t-L}$ , the inventory manager decides on (i) whether or not to place an order from outside supplier,  $w_t \geq 0$  and (ii) how to allocate the incoming order  $w_{t-L}$  among the retailers. Each of these allocations,  $z_t^j$ , arrives after an exogenously specified fixed lead time  $l$ . Hence the inventory manager should protect the retailers against the unobserved part of the demand that is to prevail during the next  $l+1$  periods, i.e. over periods  $\{t, t+1, \dots, t+l\}$ . We refer to these periods as the protection period. Notice that we can divide the protection period demand into two parts: The observed part  $O_t^{j,l} = \sum_{s=t}^{t+l} \sigma_{t,s}^j$  and the unobserved part  $U_t^{j,l} = \sum_{s=t}^{t+l} u_{t,s}^j$ . The expected holding and penalty cost charged to period  $t$  is based on the net inventory (inventory on hand minus the backorders) at the end

of period  $t + l$ . Let,

$$\begin{aligned} x_t^j &= \text{modified inventory position before shipment decision is made} \\ &= I_t^j + \sum_{s=t-l}^{t-1} z_{js} - B_t^j - O_t^{j,l} \text{ for all } j \in \{1, \dots, J\} \end{aligned}$$

$$\begin{aligned} y_t^j &= \text{modified inventory position after shipment decision is made} \\ &= x_t^j + z_t^j \text{ for all } j \in \{1, \dots, J\} \end{aligned}$$

We refer to these variables as modified since they both net the observed part of the demand information for the next  $l$  periods, hence they differ from the classical definition of inventory position. The net inventory at the end of period  $t + l$  at retailer  $j$  is given by  $x_t^j + z_t^j - U_t^{j,l}$ . Thus, the expected holding and penalty cost charged to period  $t$  for retailer  $j$  is given by  $\tilde{G}_t^j(y_t^j) = \alpha^l E g_t^j(y_t^j - U_t^{j,l})$  where  $\alpha$  is the discount factor and the expectation is with respect to the unobserved part of the protection period demand.

The state space for the exact dynamic programming formulation of this problem is  $J + L + \sum_{j=1}^J (N_j - l - 1)^+$  dimensional. It is impractical to deal with such a large state space. Hence, we develop a lower bound approximation by relaxing the constraints  $y_t^j \geq x_t^j$  from the feasible action set. This relaxation is equivalent to assuming that excess inventory at one retailer can be transferred to other retailers without any cost. Also advance orders of a customer can be satisfied through the other retailers. Intuition suggests that under this relaxation all retailers collapses into a single retailer, e.g. single location problem with advance demand information. The state space of the relaxed problem will be based on aggregate quantities. Clearly the optimal solution to this relaxation will be a lower bound. We also apply the same accounting device used for single location problem to subsume the pipeline inventory. We define next the aggregate quantities

$$D_{t,s} = \sum_{j=1}^J d_{t,s}^j, \quad O_{t,s} = \sum_{j=1}^J o_{t,s}^j, \quad X_t^\Delta = \sum_{j=1}^J x_t^j + \sum_{s=t}^{t+L-1} w_{s-t}.$$

$N = \max\{N_1, \dots, N_J\}$ . This relaxation yields the following dynamic program:

$$\begin{aligned} V_t(X_t^\Delta, O_t) &= c_{0t} X_t^\Delta + \min_{Y_t^\Delta \geq X_t^\Delta} \{c_{0t} Y_t^\Delta + \alpha^L E R_{t+L}(Y_{t+L}) \\ &\quad + \alpha E V_{t+1}(X_{t+1}^\Delta, O_{t+1})\} \end{aligned} \quad (5.15)$$

where  $V_{T+1} \equiv 0$ ,  $Y_{t+L} = Y_t^\Delta - \sum_{r=t}^{t+L-1} \sum_{s=r}^{r+l+1} D_{r,s} - \sum_{s=t}^{t+L-1} O_{s,s+l+1}$  and the update for  $X_{t+1}^\Delta = Y_t^\Delta - \sum_{s=t}^{t+l+1} D_{t,s} - O_{t,t+l+1}$  and

$$R_t(Y) = \left\{ \min_{y_t} \sum_{j=1}^J G_t^j(y_t^j) : s.t. \sum_{j=1}^J y_t^j = Y \right\}. \quad (5.16)$$

We established earlier the optimality of base stock policies for single stage problems under advance demand information. Equation (5.15) has the same structure as the single location problem. Function  $R_t(Y)$  is convex and  $\lim_{|Y| \rightarrow \infty} R_t(Y) = \infty$ . It inherits these properties from  $G_t^j(y_t^j)$ . Hence, for any fixed vector  $O_t$  there exists a  $y_t^*(O_t)$ , which is defined as the smallest minimizer of function  $H_t(Y_t^\Delta, O_t) = c_{0t}Y_t^\Delta + \alpha^L ER_{t+L}(Y_{t+L}) + \alpha EV_{t+1}(X_{t+1}^\Delta, O_{t+1})$ . Thus, a base stock policy with base stock level  $y_t^*(O_t)$  is optimal for the lower bound problem.

We can now propose a heuristic. Base stock policy solves the lower bound approximation. We solve equation (5.15) to decide how much to order from an outside supplier. The aggregate order quantity then is given by  $w_t(O_t) \doteq y_t^*(O_t) - X_t^\Delta$ . Next we have to decide how to allocate the incoming order,  $w_{t-L}$ . We propose an allocation based on the solution of the following problem:

$$\min \left\{ \sum_{j=1}^J G_t^j(y_t^j) : s.t. \sum_{j=1}^J (y_t^j - x_t^j) = w_{t-L}, y_t^j \geq x_t^j, \forall j \right\} \quad (5.17)$$

which is similar to equation (5.16) with an additional constraint  $y_t^j \geq x_t^j$ . We call this allocation strategy as *myopic* since it minimizes the total expected cost of managing the retailer inventories at the end of period  $t+l$  (the period when the allocations are available for customers) and ignores the future impact of this allocation.

In our numerical study we use a greedy algorithm while evaluating the function  $R_t(Y)$  and the equation (5.17). Let  $*y_t^j$  be the minimum of  $G_t^j(\cdot)$  and  $Y_t^* \doteq \sum_{j=1}^J *y_t^j$ . The solution to the problem  $R_t(Y_t^*)$  is trivial since the constraint allows an allocation such that the minimum of  $G_t^j(\cdot)$  is attained for all  $j$ . We use this as our initial search point. To evaluate  $R_t(Y)$  given  $R_t(Y_t^*)$  we allocate the difference  $Y - Y_t^*$ , if positive, one unit at a time to the  $j$ th retailer with the smallest current value of first difference, i.e. choose the  $\min_j \{G_t^j(y+1) - G_t^j(y)\}$ . Otherwise, we reduce the amount allocated (dis-allocate) one unit at a time from the  $j$ th retailer with the largest current value of the first difference, i.e. choose the  $\max_j \{G_t^j(y) - G_t^j(y-1)\}$ . The solution to the myopic allocation, see equation (5.17), is similar. In this case we start with

$y_t^i = x_t^i$  and allocate one unit at a time to the  $j$ th retailer with the smallest current value of the first difference until all  $w_{t-L}$  is allocated.

**3.4.2 Numerical study.** The computational study focuses on identical retailers, whose demand distribution and cost parameters are equal and stationary. We compare the solution of the lower bound (LB) problem to the solution of the proposed heuristic (UB) for different instances of the model. We report the difference as percentage error,  $\epsilon\% = (UB - LB)/LB$ , which is a measure of the sub-optimality for the proposed heuristic.

To solve the lower bound problem given in equation (5.15), we use a backward induction algorithm as in the single location problem and obtain the base stock level  $y_t(O_t)$ . We model the components of the demand vector as Poisson random variables, specifically  $d_{t,t+n}^j$  is Poisson with mean  $\lambda_n^j$ . The mean  $\lambda_n^j$  represents the average number of customers who place their orders during time period  $t$  to be delivered  $n$  periods later. We simulate the system to estimate the cost under the proposed heuristic. We run several replications of each instance to have small sampling errors and sufficiently narrow confidence intervals.

Over 84 simulations, the maximum error was 3.35%, the minimum error was 0.5% and the average was 1.48%. Our numerical study also indicates that optimality gap is insensitive with respect to the number of retailers and the lead times, see Özer [33] for more numerical examples.

One benefit of advance demand information is the resulting decline in inventory levels and inventory related costs. In Table 5.4, the percentage decrease in cost due to advance demand information between the first experiment (in which none of the customers place orders in advance) and the fifth experiment (in which all customers place orders 3 periods in advance) is 13.15%. Notice also that the system maintains lower inventory levels as we incorporate advance demand information. This

Table 5.4. Cost of LB and UB for two retailers with  $L = 1, l = 2$

$h = 1, p = 19, c = 10$		Lower Bound		Simulation	$\frac{UB-LB}{LB}\%$
No.	$(\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$	$y_t(0)$	Cost	Cost	Gap
1	(2, 0, 0, 0, 0)	25	5218.43	5273.58 $\pm$ 13.04	1.06%
2	(0, 2, 0, 0, 0)	20	5122.59	5201.65 $\pm$ 13.46	1.54%
3	(0, 0, 2, 0, 0)	14	5023.45	5110.63 $\pm$ 14.49	1.74%
4	(0, 0, 0, 2, 0)	8	4862.93	5017.27 $\pm$ 15.82	3.17%
5	(0, 0, 0, 0, 2)	0	4612.00	4646.28 $\pm$ 16.05	0.74%

$\pm$  is based on 95% confidence interval

suggests that advance demand information enables a fundamental shift in production philosophy from build-to-stock to build-to-order.

This distribution system can also be interpreted as a multi-item production system with a common intermediate product. The depot in this case represents the differentiation point. During the first  $L$  periods a common batch is produced. At the end of this period the manager has to decide how much of each item to produce and it takes  $l$  periods to produce each of these units. In our numerical analysis we also address the impact of advance demand information to postponement strategies. We do this by fixing the information horizon and the total production time to 3 periods  $N = 3, L + l = 3$ . Each of the columns in Table 5.5 represent a different differentiation point. As we move from the first column (in which we have immediate differentiation) to the third (in which we do not differentiate till the last period), we observe a reduction in both the base stock levels and the cost of managing this system. Each of the rows represent a different advance demand information scenario. In terms of reduction in inventory levels and inventory related costs, the impact of advance demand information is more significant than the postponement strategies, which are more difficult to implement. The last column of Table 5.5 shows that cost reduction due to postponement strategies is higher as we incorporate more advance demand information. Advance demand information enhances the outcome of a postponement strategy.

**3.4.3 Warehouse as stocking point.** If the warehouse is allowed to hold inventories then similar relaxation approach discussed previously yields a two stage serial system under advance demand information. In previous sections we incorporated advance demand information and established the optimality of state dependent echelon base stock policy for multi-echelons in series. We also provided an algorithm to calculate the echelon state dependent base stock levels for each stage.

Table 5.5. Advance demand information versus postponement

$(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$	$y_t(0)$			Cost of LB, $V_0$			%
	$L=0$	$L=1$	$L=2$	$L=0$	$L=1$	$L=2$	
	$l=3$	$l=2$	$l=1$	$l=3$	$l=2$	$l=1$	
(1, 0, 0, 0)	14	13	13	100.00	99.03	98.02	2.02%
(0, 1, 0, 0)	10	10	10	97.36	96.30	95.01	2.47%
(0, 0, 1, 0)	8	7	7	91.37	93.10	91.25	3.42%
(0, 0, 0, 1)	4	4	4	90.91	88.47	87.76	3.59%

$N = 3, h = 1, c = 10, p = 9$  and Index 100.00 corresponds to 2809.94

The solution to this relaxed problem yields an optimal base stock level for the first stage, which can be used to decide how much and when to order from outside supplier. Likewise, base stock level for the second stage can be used to decide on how much to withdraw from the warehouse. We would continue to allocate based on the solution of equation (5.17). We conjecture that a numerical study will illustrate that this heuristic is also close-to-optimal.

#### 4. Directions for Future Research

Research on the optimal use of demand information in supply chains is in its infancy and there are many fertile opportunities for research in this area. For single location models, research opportunities exist whenever the inventory manager has an option whose realization is based on the demand stream. We mentioned the models by Scarf and by Gallego as two examples of this idea. Additional research opportunities include the option of purchasing through futures markets, and for finite capacity manufacturers the option of selling capacity in futures markets. These ideas can also be extended to multi-echelon settings.

For multi-location models using current demand information under distributed decision making, there are research opportunities even for the serial system with two echelons. For discrete time models, it is frequently assumed that the retailer can instantaneously procure from a different source when the supplier is out of stock. Relaxing this assumption may lead to different results regarding the willingness of the retailer to share demand information with the supplier.

With regard to advance demand information, the most obvious opportunity is in distributed decision making. The simplest model would include a supplier and a retailer where the retailer obtains advance demand information. Should the retailer share this information with the supplier? If so, under what conditions? If not, should the supplier buy this information from the retailer? If so, under what conditions? Additional research opportunities exist for serial systems with more than two echelons, assembly systems, and distribution systems with multiple retailers.

#### Notes

1. Hariharan and Zipkin [26] coined the term "demand leadtime." A customer who places an order  $l$  units of time ahead of his needs is said to have demand leadtime  $l$ .
2. A function  $f$  is  $K$ -concave, if  $-f$  is  $K$ -convex
3. Under a  $(Q, r)$  policy the retailer places an order of size  $Q$  whenever the inventory position drops to  $r$ . For Poisson demands,  $(Q, r)$  policies are equivalent to  $(s, S)$  policies with  $s = r$  and  $S = r + Q$ .

4. A function  $f : \mathcal{R} \times \mathcal{R}^n \rightarrow \mathcal{R}$  is said to have decreasing differences in  $(x, \theta)$  if it satisfies the following inequality:  $f(x_1, \theta) - f(x_2, \theta) \leq f(x_1, \theta') - f(x_2, \theta')$  for all  $x_1 \geq x_2$  and  $\theta \geq \theta'$ .
5. Inventory position that nets the known requirements.

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